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The mathematics pipeline in England

Patterns, interventions and excellence

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March 2023

About this report

This report from the Mathematics Pipeline Project (2021-23) presents a system-level overview of the mathematics pipeline in England for all young people in schools from age 4 to 16 and, thereafter, a diminishing number of students who proceed to study mathematics at A level, undergraduate and postgraduate level. The project was particularly interested in those students who have the potential to remain in the mathematics pipeline into advanced and higher education, what is termed the *excellence stream* in the report.

The goal of the project was to better understand and visualise the whole pipeline and to identify areas where well-designed interventions might help to improve flow and diversity within the *excellence stream*. Whilst systemic change offers the potential for greatest impact, that was beyond the remit of the project. Rather, the report offers insights and ideas to individuals and organisations that might be interested in developing, or investing in, targeted interventions to improve the mathematics *excellence stream* in England.

The project team is grateful for the support of XTX Markets for funding this research. The views expressed within are wholly those of the authors and not necessarily those of the funder. The engagement of sector stakeholders, and the generous feedback from reviewers of earlier versions of this report, is also much appreciated.

Note: This work was produced using statistical data accessed via the ONS Secure Research Service. The use of this data in this work does not imply the endorsement of the ONS in relation to the interpretation or analysis of the statistical data. This work uses research datasets which may not exactly reproduce National Statistics aggregates.

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Executive summary

The mathematics education pipeline in England is long and complex. It comprises several million students in tens of thousands of schools, colleges and universities who are taught by over a quarter of a million teachers and lecturers. Throughout this pipeline, mathematical engagement, progress, attainment and post-compulsory participation are patterned in interesting ways; patterns which are often longstanding and stubbornly resistant to efforts to ameliorate them.

This report is particularly interested in what we term the *excellence stream;* that part of the mathematics pipeline which includes students with the capabilities to progress to advanced level and university mathematics. This *excellence stream* diminishes over time across the phases of education and, importantly, with different rates for students from diverse backgrounds. Of particular concern in this project are the trajectories of students from the most disadvantaged backgrounds.

The report presents a 'helicopter view' of the flow patterns of students through the mathematics pipeline (Section 2). It discusses the key features of these patterns and synthesises research/expert insights into their causes (Section 3). This is followed by an examination of the current landscape of interventions in mathematics education (Sections 4) and some considerations of possible new avenues for innovations and interventions (Section 5).

Four important features of the pipeline are noteworthy:

- **The Key Stage 3 attainment gap** opens up following transition to secondary school. Many students from economically disadvantaged backgrounds are thereby lost from the mathematics *excellence stream* through to GCSE. For those who remain, and attain highly at GCSE, they are as likely to progress to A level Mathematics as their more affluent peers;
- **Asian students' mathematics progress** through secondary schooling, and their subsequent attainment and participation in advanced mathematics is striking in its positive divergence from the trends of other ethnic groups;
- **Girls' participation in A level Mathematics** continues to be lower than that of boys (and even more so in Further Mathematics) and this trend continues into undergraduate study in mathematics. Ameliorating girls' loss from the mathematics *excellence stream* needs new approaches earlier in secondary education, and before that.
- In the **transition to university** economically disadvantaged students are more likely than their affluent peers with similar prior attainment to progress to undergraduate mathematics. However, once at university, those same students are less likely to complete their mathematics degree. In addition, many 18-year-olds are lost from the mathematics *excellence stream* to the life and social sciences.

Influencing choice patterns into A level, undergraduate and postgraduate study happens in multiple ways and at every educational stage. Holistic understanding of the genesis of those choice patterns is needed to inform better interventions that improve engagement, progression and attainment (e.g. at 16) and thereafter participation in mathematics. This is a long-term project, made more challenging when the societal and political influences on the mathematics pipeline change so frequently. However, as with all complex systems, it is not clear what combinations of actions will yield sustained changes in choice patterns and improvements in the pipeline, nor indeed which will unintentionally disrupt the flow.

Designing effective interventions, initiatives and policies is notoriously challenging, though there is growing international expertise in this area. Similarly, the need for robust evaluations of interventions that can help to build the knowledge base is important. All of this is easy to state, as many have done, and yet difficult to achieve.

Given the interests of the funder of this research, the report culminates in the identification of key areas for action and possible interventions in Section 5. Briefly, these are as follows:

1. **Improving engagement and progression in Key Stage 3**
   - Specialist teacher programmes.
   - Maths clubs and competitions.
   - Virtual maths schools.
2. **Improving participation in mathematics beyond 16**
   - Signalling and career promotion programmes.
   - Innovations to tackle social/cultural barriers.
   - Becoming a mathematician projects.
3. **Coordinating interventions**

The report concludes by setting out some general principles about intervention development and implementation, together with the need for coordinated systematic evaluation and longitudinal research that can inform a more sustainable change agenda and which contributes to improving the mathematics pipeline generally and the *excellence stream* in particular.

Improving the mathematics education pipeline for all is a major undertaking and is beyond the remit of this report, though much of what is written here will be of relevance to those tasked with orchestrating such improvement. The project scope was not primarily concerned with national policy such as curriculum and assessment, teacher supply, professional development landscapes and the like, but rather aimed to identify areas for 'high leverage' interventions in which motivated individuals and organisations might focus their energy and resources in order to improve an aspect of the mathematics pipeline, in particular the *excellence stream*.

March 2023

1. Mathematics education in England: an introduction

1.1 Why the mathematics pipeline matters

Evidence for the personal, social and economic benefits of mathematics has generated broad and sustained interest in improving mathematics education in advanced economies. In England, the importance of mathematics is evidenced in a plethora of reports on everything from everyday numeracy (National Numeracy, 2019) to the contributions of mathematics research (Bond, 2017; Deloitte, 2012). Calls to improve mathematics education are fuelled by international comparisons of education systems[1], concerns about the future of the world economy and our competitiveness within it, and a commitment to 'level up' society. Such matters have arguably never been more pressing.

The general importance and widespread applicability of mathematical and data sciences and of quantitative and statistical literacy means that subtly different purposes for mathematics education come to the fore at different times, and are championed by interested stakeholders. Appreciation of these drivers, and their affordances and constraints for policy and practice is needed. Three broad purposes for mathematics education that reflect this plurality are:

- **Mathematics for employment**. Applications of mathematics at all levels and in every area of science and the economy require diverse procedural and problem solving skills together with so-called techno-mathematical literacies (Hoyles et al., 2010). Repeated calls from business and industry for improved skills, together with high-level analysis of the links between mathematical competences and economic productivity (Hanushek et al., 2013) strengthen this drive to increase human capital through mathematics education.
- **Mathematics for its own sake.** Pursuing new mathematical knowledge and appreciating the cultural impact of mathematics on humanity across the millennia are particularly important for some educators. Learning mathematics for its own sake can be overshadowed by the utilitarian 'skills' and 'literacies' drivers above, though in many classrooms and university lecture theatres more humanistic goals are still very much alive. Such examples of mathematics education focus less on employability, economic returns and critical citizenship but rather take pleasure in the abstractions of mathematics itself.
- **Mathematics for citizenship**. Mathematical skills[2] predict health outcomes and improve financial decision making, democratic participation and the ability to spot misinformation, amongst other things. Developing competences to understand, and act responsibly with respect to issues such as public health and environmental concerns is increasingly important. Enhancing such capabilities for 'reading the world with mathematics' (Gutstein, 2012) is a priority for some mathematics educators.

Policymakers, leaders, teachers and other stakeholders are motivated by different combinations of these purposes which in turn shapes the policies, initiatives, interventions and practices that crowd the mathematics education landscape. These purposes are also not mutually exclusive; educationalists are often pluralists, seeing merit in combinations of these purposes for different students and on different

[1] For example, the Programme for international Student Assessment (OECD: https://www.oecd.org/pisa/); Trends in Mathematics and Science Study (TIMSS: https://timssandpirls.bc.edu/timss-landing.html); the Programme for the International Assessment of Adult Competences (PIACC, https://www.oecd.org/skills/piaac/)

[2] The terms numeracy, quantitative literacy and data literacy could be substituted here.

occasions. Debates and contestations about mathematics curricula and high-stakes qualifications surface repeatedly, though the values and purposes underpinning different positions are not always made explicit.

This report explores students'[3] progress in mathematics in England, from the start of reception (age 4) to the end of Key Stage 4 (age 16) and thereafter into sixth forms, colleges and universities; the so called 'mathematics pipeline'. Of particular concern herein are those students who have the potential to progress to undergraduate and post-graduate mathematical study and thereafter into research, what might be termed the *excellence stream*. The diversification of this *excellence stream*, particularly from underrepresented groups, is also of interest in this report.

The metaphoric language of pipeline[4] is associated with flow volumes and rates, and with leaks (van den Hurk et al., 2019). It would be tempting to focus attention on the points where the mathematics pipeline narrows or 'leaks' most dramatically (i.e. age 16 and 18 in England). These are of course critically important points for action, and others have already explored these contractions in the pipeline (e.g. Hodgen, Pepper et al., 2010). Yet a focus on leaks is limiting as the evidence is clear that many young people have effectively drifted out of, or been filtered from, the *excellence stream* well before the age of 16. This leaves them unqualified for, or unenthusiastic about, progression to advanced mathematics.

This report takes a long, high-level view of the mathematics pipeline from reception to postgraduate study and research. This holistic perspective brings together two views that often remain disconnected: that from schools (to 18) and from universities (from 18). Before that, it is instructive to set out some of the high-level context of mathematics education in England[5].

1.2 Some high-level features of the pipeline

There are now over nine million students in around 24,400 schools in England. They are taught mathematics by over a quarter of a million primary teachers and around 35,000 secondary teachers.[6] The budget for England's maintained schools is over £43 billion, and mathematics education can lay claim to a good proportion of this[7].

School students generally follow the national curriculum for mathematics which, in the high-stakes assessment and accountability culture in which England's schools operate, has resulted in some criticism of overly procedural mathematics learning. Although attainment in GCSE at age 16 has improved over the years, there are still far too many students leaving school with poor attitudes to mathematics and insufficient quantitative skills for the demands of modern workplaces and personal decision-making. Furthermore, academics observe that new undergraduates don't always think like mathematicians.

[3] Multiple terms are used to describe learners in the reception-postgraduate pipeline. The report adopts the term 'students' throughout, rather than switching between children, pupils, young people and/or learners.
[4] For a discussion of the pros and cons of the metaphors of pipeline, pathways and participation in relation to mathematics see Noyes and Adkins (2016).
[5] A helpful short comparison with the other nations of the UK can be found at https://www.jmc.org.uk/2020/07/02/mathematics-education-5-18-in-the-four-nations-a-comparative-report/
[6] https://www.gov.uk/government/statistics/school-workforce-in-england-november-2021. 35,000 is an overestimate of those spending most or all of their time teaching mathematics, which is arguably nearer to 25,000; still a large number.
[7] This does not include the considerable sums spent in private and shadow education. For a recent discussion of this see https://www.suttontrust.com/wp-content/uploads/2019/12/Shadow-Schooling-formatted-report_FINAL.pdf

For some time, there has been a national policy priority on 'teaching for mastery', in particular in the primary and lower secondary years. This has been facilitated through a network of Maths Hubs[8], 32 of which were established in 2014. There are now 40. Together with the National Centre for Excellence in Teaching Mathematics (NCETM), the hubs represent a major investment in mathematics education and teacher professional development.

Reforms to GCSE Mathematics during the last decade were designed to increase the mathematical level of demand[9] yet the jump from GCSE to A level Mathematics remains considerable. Many learners cease their focused study of mathematics at age 16 and only the highest attaining GCSE students go on to achieve the top grades at A level. The 2017 reforms to A level Mathematics established a common curriculum (including statistics and mechanics) and a new emphasis on problem solving and modelling. Parallel to this, the introduction of Core Maths has introduced a new post-16 mathematics pathway, albeit uptake remains modest to date[10].

Government investment in the Advanced Mathematics Support Programme (and its predecessors the Further Maths Support Programme[11] and Core Maths Support Programme) reflects a sustained political commitment to mathematical excellence at advanced level and to increasing post-16 participation. A level numbers have generally risen over the last 10 years[12], albeit with some impact at the time of the qualification reforms, yet despite sustained calls for mathematics for all to 18[13] this remains some way off. Enacting a successful policy of mathematics for all to 18 would be contingent upon addressing concerns over qualifications, teacher capacity and professional development, and student motivation.

Around 7000[14] students progress from schools and colleges in England to undergraduate studies in mathematics each year. Interestingly, this number has remained fairly constant despite the growth in A level entries. At the same time, there are fresh concerns in the mathematics learned societies around the expansion of mathematics cohorts in research-intensive universities that threaten undergraduate courses in some institutions.

A few leading universities have supported a recent policy of establishing specialist maths schools[15]. These have yet to establish their place in the mathematics education landscape but they signal the Government's commitment to mathematical excellence. Maths schools are one type of innovation in the evolving school landscape and any effort to effectively intervene in mathematics education in England needs to be cognisant of the wider system; mathematics education does not exist in isolation. Scholars and commentators have noted the considerable shifts in the education system in England in recent decades; an ongoing process of fragmentation and reformation as Local Authorities are rolled back and new organisational structures take centre stage.

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[8] https://www.ncetm.org.uk/maths-hubs/

[9] For example, see https://www.gov.uk/government/speeches/oral-statement-on-education-reform

[10] https://eprints.whiterose.ac.uk/168941/1/PolicyLeeds-Note1_Core-Maths.pdf

[11] The FMSP followed the disastrous impact of the Curriculum 2000 reforms which saw an exodus from A level mathematics and widespread concerns about how to redress this.

[12] From 2012-22 candidate numbers for A level Mathematics rose from 78,950 to 88,315, a 12% increase, though the increase for males (21.4%) was significantly higher than for females (5.2%) [See https://analytics.ofqual.gov.uk/apps/Alevel/Outcomes/]

[13] Michael Gove's speech to the Royal Society (https://www.gov.uk/government/speeches/michael-gove-speaks-to-the-royal-society-on-maths-and-science) "I think we should set a new goal for the education system so that within a decade the vast majority of pupils are studying maths right through to the age of 18"; See also Smith (2017) and Sunak (2023, https://www.gov.uk/government/news/prime-minister-sets-ambition-of-maths-to-18-in-speech)

[14] www.ucas.com/data-and-analysis/undergraduate-statistics-and-reports/statistical-releases-daily-clearing-analysis-2021

[15] https://www.gov.uk/government/publications/how-to-open-a-maths-school

In 2022 the Government set out an ambition[16] that by 2030 all schools should be in, or joining, a strong trust. Currently there are around 1500 Multi Academy Trusts (MATs) in England but this is an evolving landscape[17]. Some of the larger MATs have established well-organised oversight of their mathematics provision. How these MATs coordinate with the Maths Hubs varies, and the future of the various Hubs when all MATs are of sufficient size to provide a patch-worked oversight of the whole education system in England is moot.

A young person's educational career lasts around as long as 7-10 Secretaries of State[18], each of whom seek to bring positive change to the education system with new policies and reforms. It is notoriously difficult to evaluate robustly the impact of any one change in the system, and virtually impossible to understand the impact upon students of multiple and sustained changes over a full educational career. The challenge of coordination is clear, whether of policy implementation in institutions or of orchestrating the many interventions[19] of third-sector organisations, edu-businesses and the like across schools, trusts and other networks. The problem of who has oversight, and coordination responsibility[20], is discussed below.

1.3 The report

The report is organised as follows. Section 2 overviews some of the important patterns in the mathematics pipeline, zooming in on some key transition points and phases. There are many interesting areas for discussion which are beyond the scope of this report, but a focus on economic disadvantage is included.

In Section 3, the report proceeds to a high-level synthesis of what is known about the mathematics pipeline in England, based on analysis of the research and policy literatures. Additionally, this section draws on interviews with a range of experts in mathematics education from primary to higher education, from civil servants to representatives of philanthropic organisations, and either working in education generally or mathematics education specifically.

In the final section of the report (Section 4), we discuss interventions and initiatives in mathematics education and propose a typology that can help to explore the affordances and constraints of various interventions. The typology would be useful for those designing future interventions by challenging them to answer key questions about what can, and cannot, be achieved given their design parameters.

The report culminates in Section 5 with some discussion of promising areas/foci for new interventions and suggestions for future research. These are based on the integration of insights from the project and, importantly, from the team's collective insider knowledge of the educational system at primary, secondary and higher education levels. It is important that the implementability of any new intervention is considered carefully and, as such, thought is given throughout to how any new interventions might be mapped onto the current and emerging educational structures.

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[16] https://www.gov.uk/government/publications/opportunity-for-all-strong-schools-with-great-teachers-for-your-child. Although the government has since changed its position on the policy as a whole in early 2023, the likelihood of a move to a self-improving MAT-organised system seems very likely.

[17] https://ffteducationdatalab.org.uk/2022/05/the-size-of-multi-academy-trusts/

[18] A student who started reception in 2006 and is due to graduate in summer 2023 will have had their education overseen by at least 11 Secretaries of State.

[19] The term intervention should be understood broadly to describe additional efforts – beyond the typical educational offer - that address perceived gaps/needs in mathematics education. These might be enacted at national, local or student scales.

[20] The Wellcome-funded EQuaLLS project is exploring some of the challenges of local coordination of professional learning opportunities for primary teachers of mathematics: https://equalls.uk/

2. Patterns in the mathematics pipeline

The full mathematics education pipeline is around 20 years long, from the start of reception aged 4 through to post-16, undergraduate and postgraduate study choices, and perhaps research thereafter. Surveying the full pipeline at one point in time conflates the impact of different historical states and developments. For example, current doctoral researchers (educated in England) experienced the National Numeracy Strategy at primary school whereas today, primary pupils are likely to experience mathematics lessons based on some version of a mastery approach.

To understand the dynamics within the pipeline, a combination of approaches is necessary. One might focus on specific points in the pipeline (e.g. A level outcomes) and monitor them over multiple time points; something akin to viewing a river flow from a bridge. Another approach could be to track specific students or cohorts over time; as if flowing along the river. Neither strategy is sufficient on its own to understand fully the current state of the mathematics pipeline or to inform new intervention strategies. We follow the latter approach here, incorporating elements of the 'bridge' view by using multiple cohorts.

2.1 Data and methodology

England boasts exceptional educational datasets including the Department for Education's National Pupil Database (NPD) and Higher Education Statistics Agency (HESA) data. To develop the best, most recent, insights into the mathematics pipeline, the research team obtained three different sets of linked cohort data. These were accessed through the Office for National Statistics Secure Research Service.

Cohort 1 comprises a full, national cohort of students in the NPD who took GCSEs in the academic year 2016/17, linked to their academic records back to the end of Key Stage 1 (in 2007/08) and forward to A levels (in 2018/19). This cohort was chosen to capture the most recent cohort to have typically completed A levels, but prior to the impact of the Covid-19 pandemic. Analysis of cohort 1 gives insights into patterns of progress and attainment for different student characteristics in the cohort over time, as well as participation trends at A level[21]. This cohort was the first to experience both the new grading system at GCSE (9-1) and the reformed A level Mathematics qualification, both of which have increased focus on problem solving and modelling and with greater focus on terminal assessment. Their primary schooling was heavily influenced by the Primary Framework for Literacy and Mathematics and at KS3, the Secondary National Strategy (SNS).

Cohort 2 comprises a base HESA cohort of around a quarter of a million first-year undergraduate students in 2015/16. These are linked back to their A level and GCSE outcomes and forward to undergraduate outcomes and postgraduate degree choices[22]; they might well have come from different A level cohorts, and they end up in different graduating cohorts. This cohort was selected as the most recent cohort to complete their degree programme pre-pandemic. Analysis would enable understanding of progression patterns from A levels to undergraduate study in mathematics, including where high-attaining 18-year-olds go, if not into advance study of mathematics and its applications. This group were in primary school during the mature stages of the Primary national Strategy (PNS) and the SNS, typically moving to secondary school in 2008. Their GCSE and A levels were taken prior to the

[21] Although a full national cohort is around two-thirds of a million students (at age 16), the analysis below is with those students for whom the full record from 4-16 (and beyond) is available. It should be noted that although the missing student cases are not missing at random, the diagrams still offer broadly representative and useful views of the pipeline.

[22] We only include undergraduate students who have previously taken GCSE and A levels in order to focus on students who have come through the English or Welsh education system.

qualification reforms instigated by the then Secretary of State Michael Gove. At that time A levels were still modular and allowed for some choice between the study of mechanics, statistics and 'decision' modules.

Cohort 3 (a GCSE cohort from 2008/9) enabled additional, confirmatory analysis of the full pipeline, albeit from an earlier cohort. This cohort began school in 1997, when the national curriculum, national testing and Ofsted were in their infancy. Their primary schooling coincided with the energetic 'education, education, education' policies of Tony Blair and David Blunkett and the sharp rise in primary mathematics outcomes associated with the early years of the National Numeracy Strategy[23]. They were the last cohort to complete end of Key Stage 3 tests which afforded the team the opportunity to explore attrition from the *excellence stream* between ages 11-14. This third cohort also allowed for some of the analysis of changing cohort characteristics over time reported below. In practice, most of what is reported briefly below was derived from analysis of Cohorts 1 and 2.

For each cohort, the student data includes their mathematics national test or examination outcomes[24]. For Cohort 1, these are for the end of Key Stages 1, 2, 4 and 5; for Cohort 2 they are at end of Key Stages 4, 5 and their undergraduate degree subject. Cohort 3 includes data from the end of all Key Stages (including the end of Key Stage 3) and their undergraduate[25] and postgraduate degree choices.

Each dataset includes demographic information such as gender[26], ethnicity and social class[27] as measured by Income Deprivation Affecting Children Index (IDACI) quintile[28] at the end of Key Stage 4 (IDACI quintile 1 are the most deprived students and quintile 5 are the least deprived students). Whilst the IDACI measure is fine grain it is based on a student's postcode which therefore includes multiple households and potentially homogenises quite different families. In contrast, eligibility for Free School Meals (FSM) is specific to the individual but is a binary measure.

2.2 Overview of the pipeline

This section begins by presenting a high-level view of the pipeline by focusing on the progression from the start of school through to A levels, and then to undergraduate study before considering the pipeline characteristics more generally. It highlights groups of students, and stages of the pipeline, that might benefit from interventions in order to retain a greater number of students in the *excellence stream*.

2.2.1 Progression from Key Stage 1 to A level Mathematics

Figure 1 shows the flow of Cohort 1 students through the mathematics pipeline from Key Stage 1 to A level. 19.7% of these students achieved grades 7-9 at GCSE with 3.5% achieving grade 9. At A level, 9.4% of this cohort took single Mathematics and a further 1.7% took both A level Mathematics and Further Mathematics. By tracing the routes of students who ended up taking A level Mathematics, and who therefore had the opportunity to study mathematics at university, one can see where students need

[23] The National Numeracy Strategy's Framework for Teaching Mathematics reception to Year 6 was implemented in 1999.
[24] The GCSE grading system changed between Cohort 1 (grades 9-1) and Cohort 2 (grades A*-G). Grades 7-9 are broadly equivalent to grades A* and A. Where a student has attempted a qualification on more than one occasion then we use the highest grade they achieved. We use notation such as 3+ to denote (national curriculum) levels or GCSE grades 3 or above, and similarly 3- and 6- to denote levels or grades 3 or 6 or below respectively. For the end of Key Stage 2, in order to give greater granularity in the analysis, we use raw scores to divide level 5 into level 5U and level 5L, to denote grades at the upper and lower end of level 5.
[25] Undergraduate subject choices are based on the JACS3.0 (2015/16-2018/19) and CAH-01 (2019/20) classification system. A student is deemed to be studying mathematics if their course classification is 100% CAH09 (mathematical sciences) or joint mathematics if mathematical sciences comprises more than 30% of their course classification.
[26] The NPD uses the label *gender*, rather than *sex*, to indicate whether a student is male or female.
[27] Measures of social deprivation are taken in 2016/17 for Cohort 1 and 2008/9 for Cohort 2.
[28] Note that IDACI quintile 1 is the one with the greatest income deprivation (i.e. the poorest).

to be at each stage to stay in the *excellence stream*. Those who took A level Mathematics, or both A level Mathematics and Further Mathematics, almost exclusively had grades 7-9 at GCSE and achieved level 5 or above at the end of Key Stage 2. Even mathematics performance at the end of Key Stage 1 is a reasonable predictor of A level participation, with 30.3% of those who achieved level 3 progressing to A level Mathematics compared to just 1.3% of those who achieved level 2C or below[29].

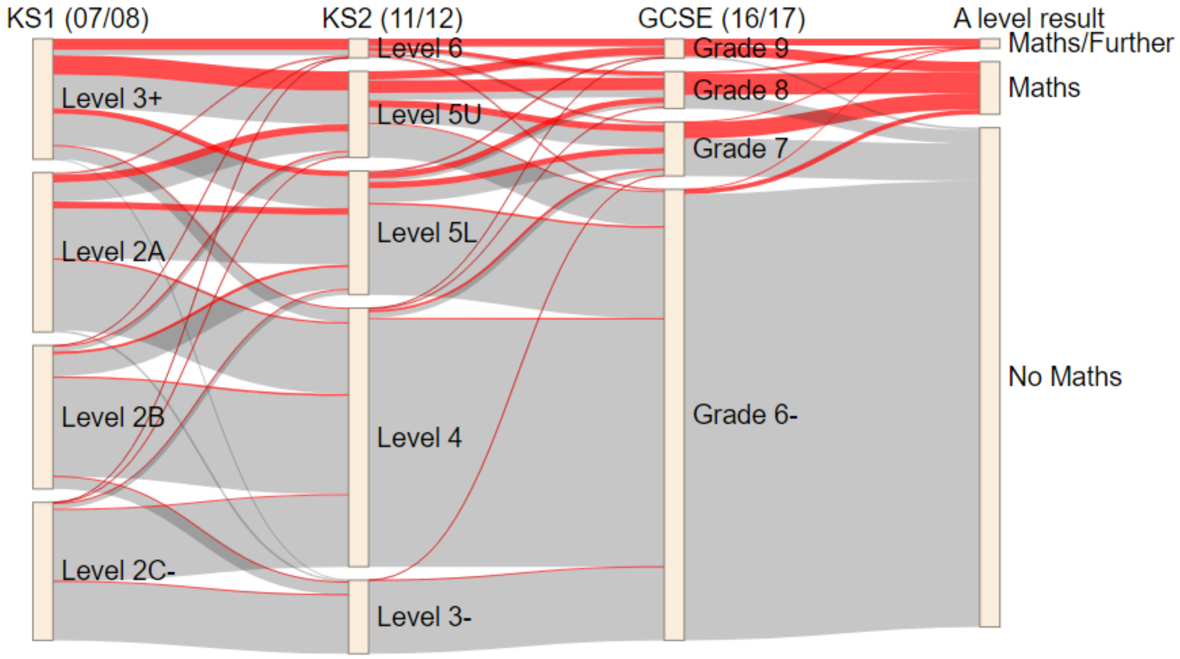

Figure 1: Sankey diagram summarising (Cohort 1) student flows from the end of Key Stage 1 (aged 6/7 in 2007/8) to A level (2018/19), with pathlines to A level Mathematics and Further Mathematics highlighted. 'No maths' indicates those students not completing advanced mathematics qualifications by this point and includes 'studied maths but did not complete' and 'studied A-levels but not maths'. NB Level 5 at Key Stage 2 has been divided into upper (5U) and lower (5L) outcomes using the fine grain results, in order to identify a group that approximately represent the top fifth of the cohort (6/5U)

As well as observing that the students with the lowest grades at the end of each Key Stage have a small chance of progressing to A level, it is worth noting that 12.0% with GCSE grade 9 and 36.3% with grade 8 did not progress to complete A level Mathematics. The composition of the advanced mathematics contingent is determined either by a) students being filtered out due to low prior attainment and/or b) students choosing other study options.

2.2.2 Progression to undergraduate mathematics study

Figure 2 represents a full undergraduate cohort of students starting in 2015/16. It shows the prior pathways through GCSEs and A levels of this group, 2% of whom embarked on a degree entirely within the mathematical sciences, and a further 1% a joint degree programme (i.e. in which mathematics modules comprise at least 30%). Combining single and joint degree programmes, 81% of students who started these degrees completed them, whereas 8% moved to a degree programme with little or no mathematics and 11% left with no qualification[30].

[29] The equivalent percentages are 11.0% for level 2A and 4.4% for level 2B. Of the students who studied A level Mathematics 30,890 had achieved level 3 or above at Key Stage 1, 14780 had achieved level 2A, 5365 had achieved level 2B and 1540 had achieved level 2C or below.

[30] For comparison, across all subjects 13% of students who start a degree leave with no qualification.

As expected, GCSE and A level grades are both good predictors of completing a mathematics degree (single or joint honours). Of those students in this undergraduate cohort who achieved GCSE grade A*, 10% completed a mathematics degree compared to 3% of those with grade A; and at A level 37% of those who achieved A*A or above[31], across Mathematics and Further Mathematics, went on to complete a mathematics degree compared to 12% of those with A or A* in A level Mathematics but without Further Mathematics. 3,800 students in this cohort achieved A*A or above across Mathematics and Further Mathematics but did not complete a single or joint degree programme in mathematics, despite having the desire to study both A levels and the aptitude to achieve high grades. Many of these progressed to mathematically-demanding degree programmes (see Figure 7). Mathematics degree programmes attracted 24.5% of the students in this cohort who achieve A or A* in one or both mathematics A levels.

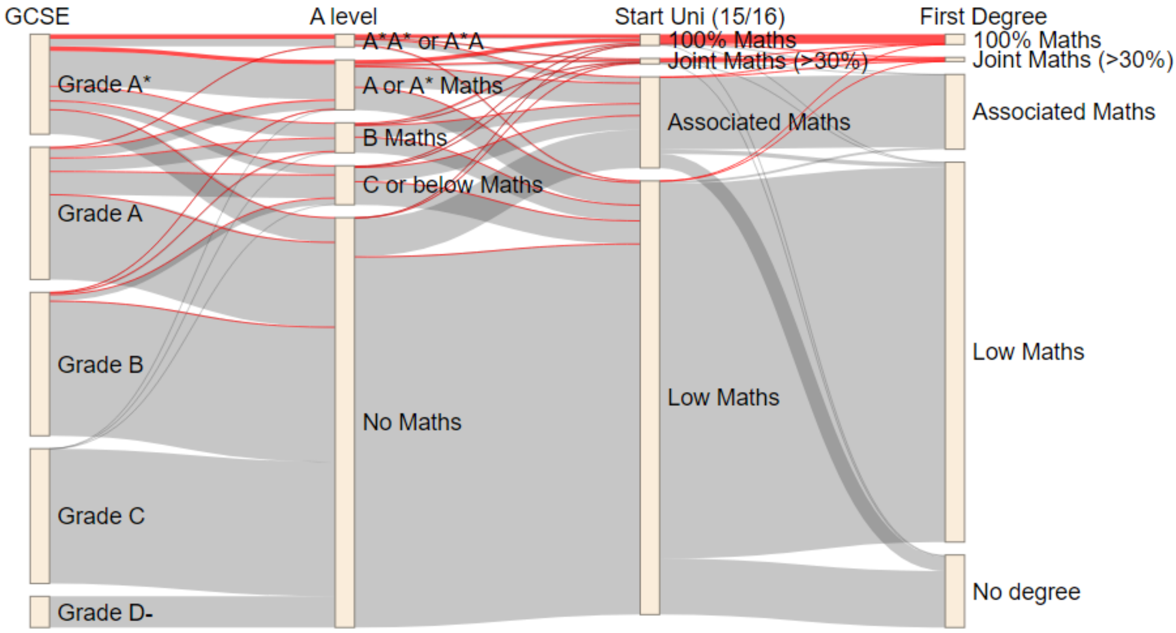

*Figure 2: Sankey diagram summarising Cohort 2 student flows from GCSE[32] to degree completion (or non-completion, i.e. 'no degree'), with pathlines to undergraduate degrees in single or joint mathematics highlighted. The highest attaining A level category in the diagram relates to students that took both A level Mathematics and Further Mathematics and achieved A*A*, A*A or AA* across both qualifications.*

2.2.3 Student characteristics in the *excellence stream*

For Cohort 1, the *excellence stream* was assumed to comprise students attaining level 3+ at the end of Key Stage 1; those awarded level 6 and the upper half of level 5 at the end of Key Stage 2, and those attaining grades 7-9 in GCSE Mathematics. For cohort 2, the *excellence stream* included students achieving A*/A in GCSE Mathematics and an A* or A in at least one of A level Mathematics or Further Mathematics.

Figures 3 a,b,c show the percentages of three student groups in Cohort 1 (gender, SES and ethnicity) who were in the *excellence stream* as described above at different stages of the pipeline, together with the percentage that completed A level Mathematics (and/or Further Mathematics). For example, Figure

[31] We focused on those attaining A*/A at A level given that this is a differentiating factor in recruitment to mathematics degrees in so called selecting universities. That is not to say that achieving other grades at A level, and then proceeding to undergraduate study, is not a worthwhile pathway. Rather, this analysis assumes A*/A grades as the threshold for the *excellence stream.*
[32] the GCSE grades for this cohort fall under the old grading system.

3a shows that 23.7% of male students in this cohort achieved level 3 or above at the end of Key Stage 1 compared to 19.0% of female students. This pattern continued through to Key Stage 2 although the gap narrowed significantly at GCSE. It widens again at A level, with 14.6% of male students and 9.6% of female students from Cohort 1 completing A level Mathematics. For the earlier Cohort 3, these figures are 12.7% and 8.9% respectively for male and female students suggesting, in line with the point made earlier[12], that A level completion has increased, although more so for males than females. For Cohort 1, the resulting A level Mathematics cohort has a gender split of 61.5% male and 38.5% female.

In terms of socio-economic status (SES), Figure 3b shows those from less deprived areas to be over-represented in the top-grades at GCSE and in the A level cohort. Those from IDACI quintile 5 are 2.6 times more likely to achieve level 3 at the end of Key Stage 1 than those in quintile 1. This ratio remains fairly constant throughout the pipeline, with those from IDACI quintile 5 being 2.7 times more likely to achieve grade 7 or above at GCSE and 2.5 times more likely to complete A level Mathematics compared to quintile 1. For the earlier Cohort 3, those from IDACI quintile 5 are 2.3 times more likely to complete A level Mathematics compared to those in quintile 1 suggesting that the SES participation gap has widened in the more recent cohort.

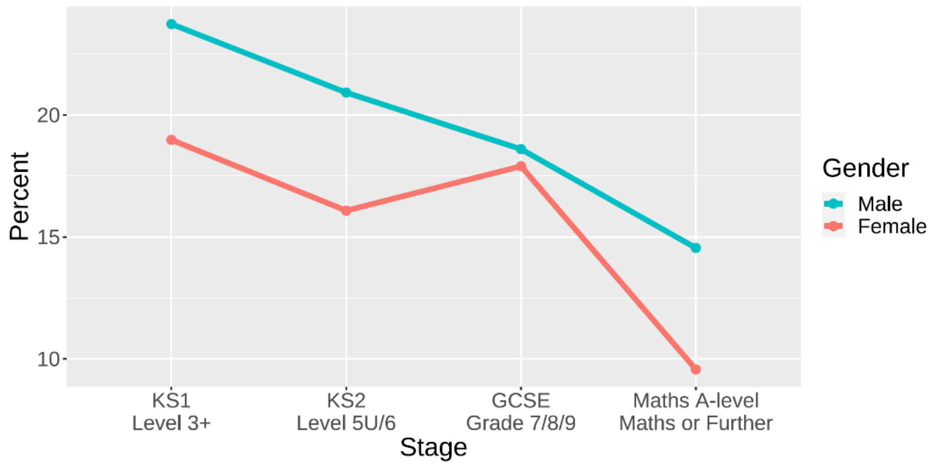

Figure 3a: The percentage of male and female students in Cohort 1 achieving level 3 or above at the end of Key Stage 1, level 5U or above at the end of Key Stage 2, grade 7 or above at GCSE and completing at least one of A level Mathematics or Further Mathematics.

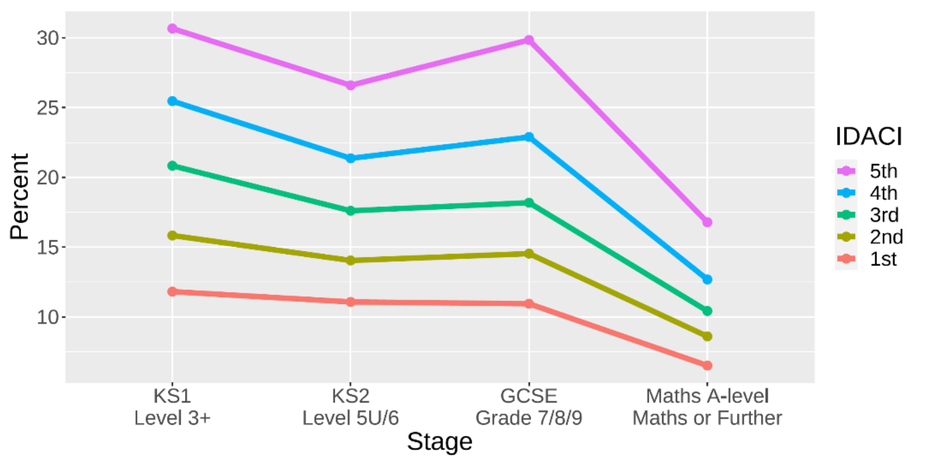

Figure 3b: The percentage of students from each IDACI quintile in Cohort 1 achieving level 3 or above at the end of Key Stage 1, level 5U or above at the end of Key Stage 2, grade 7 or above at GCSE and completing at least one of A level Mathematics or Further Mathematics.

Regarding ethnicity, Figure 3c shows that 22.1% of white students included in Cohort 1 achieved level 3 or above at the end of Key Stage 1 compared to 11.4% of black students. The proportion of black students achieving the top grades at the end of Key Stage 2 is also much less than the other three ethnic categories which are broadly similar, but the gap narrows at GCSE and A level where, in contrast, a notably higher proportion of Asian students complete A level Mathematics.

Even though only 10.6% of Cohort 1 students are Asian, they comprise 15.6% of students achieving grades 7 to 9 at GCSE and 21.2% of A level Mathematics students. Between the earlier Cohort 3 and Cohort 1 (shown in Figure 3c), the A level participation rate increased by 1.8 percentage points for black students with only modest increases for Asian and white students (up 0.2 and 0.4 percentage points, respectively). However, the A level participation rate for students with mixed ethnicity fell by 1.1 percentage points.

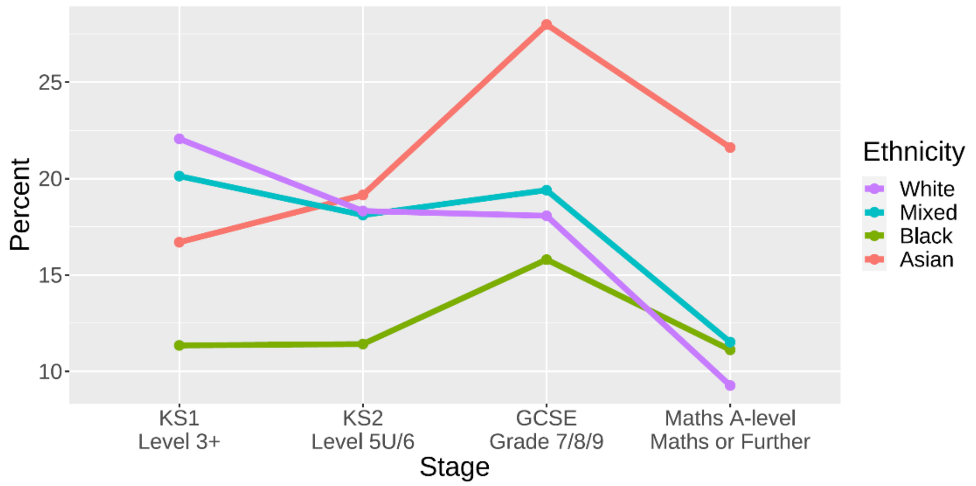

Figure 3c: The percentage of students from each ethnicity in Cohort 1 achieving level 3 or above at the end of Key Stage 1, level 5U or above at the end of Key Stage 2, grade 7 or above at GCSE and completing at least one of A level Mathematics or Further Mathematics.

Figures 4 a,b,c show the compositions of Cohort 2 during the later stages of the *excellence stream* of the mathematics pipeline. Figure 4a shows that approximately equal numbers of male and female students achieved grades A* or A at GCSE. However, different progression rates means that at A level only 37% of those achieving grades A* or A in Mathematics are female (and that proportion reduces further to 26% for Further Mathematics). The composition of the undergraduate mathematics cohort is 65% male and 35% female.

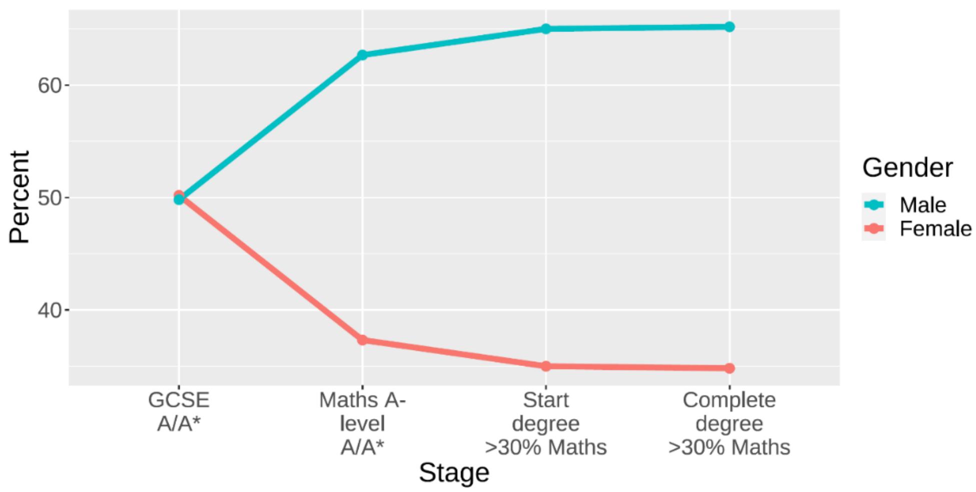

Figure 4a: The gender split of students in Cohort 2 achieving grade A or A at GCSE, A or A* in A level Mathematics, and at the start and completion of first degree level courses where mathematics comprises at least 30%.*

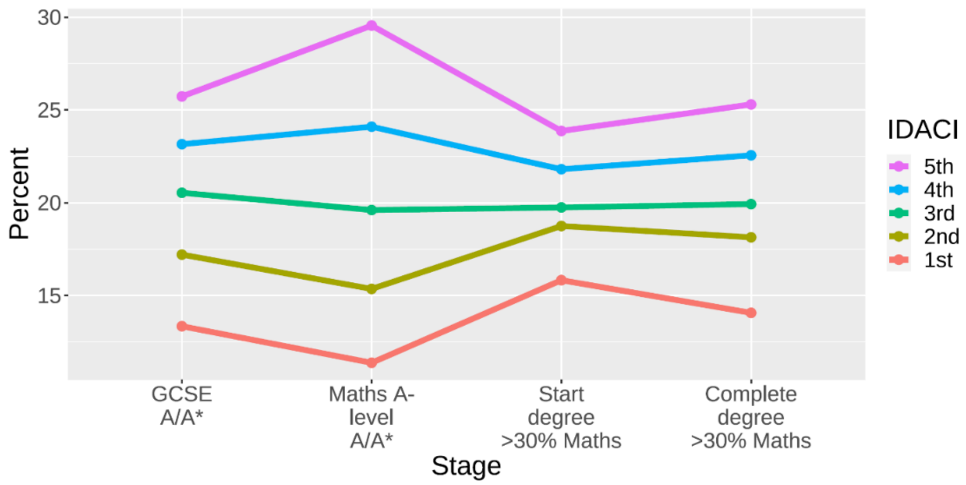

Figure 4b: The composition by IDACI quintile of students in Cohort 2 achieving grade A or A at GCSE, A or A* in A level Mathematics, and at the start and completion of first degree level courses where mathematics comprises at least 30%.*

Figure 4c shows that the ethnic composition of high achieving students at GCSE and A Level is fairly similar to the composition of the undergraduate mathematics cohort, although Asian students remain over-represented (see Figure 3c).

Some characteristics of the *excellence stream* change as students flow along the mathematics pipeline while others remain the same. For instance, girls are persistently underrepresented at every stage except at GCSE where the gender gap is at its narrowest. Similarly at every stage, the greater the deprivation, the less likely a student is to remain in the *excellence stream* with the gap between the least and most advantaged most noticeable at A level.

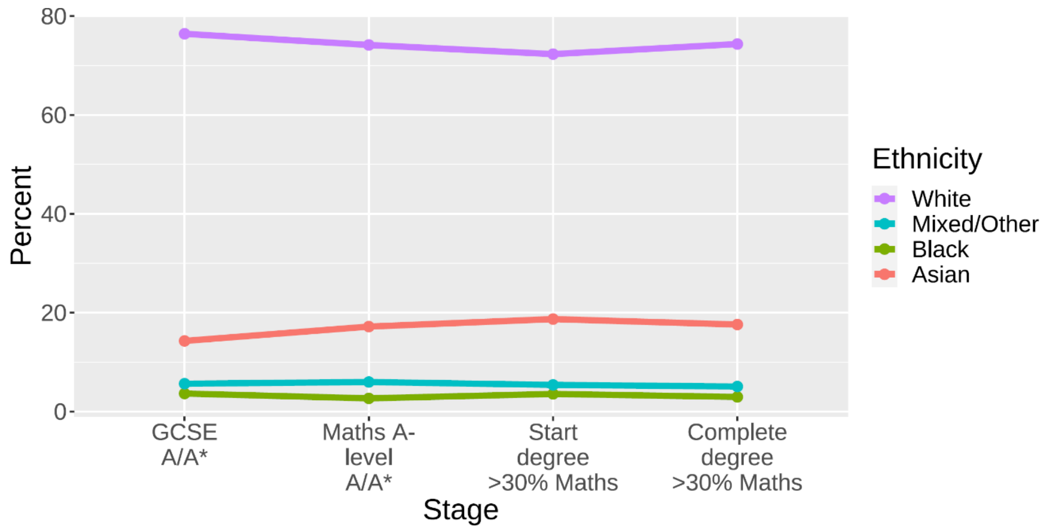

Figure 4c: The composition by ethnicity of students in Cohort 2 achieving grade A or A at GCSE, A or A* in A level Mathematics, and at the start and completion of first degree level courses where mathematics comprises at least 30%.*

2.2.4 Remaining in the *excellence stream*

The Sankey diagrams in Figures 1 and 2 clearly show that prior attainment is key to predicting who stays in the *excellence stream* at the subsequent stage. This is explored further below by tracing the proportion of students who remain in the *excellence stream* from one stage to the next. Figure 5 shows the proportion of students in Cohort 1 who achieve top grades at the end of Key Stage 2 and go on to achieve grades 7-9 at GCSE. Even though females are underrepresented in the highest grade categories at both the end of Key Stage 2, and marginally at GCSE (see Figure 3a), they have a greater chance of remaining in the *excellence stream* when controlling for prior attainment. This suggests that if the number of female students achieving high grades at the end of Key Stage 2 can be increased, this might well lead to a higher number of high attaining female students at GCSE. However, Figure 5 also shows that only 52.1% of high performing students from IDACI quintile 1 (the poorest) at the end of Key Stage 2 progress to grades 7-9 at GCSE, compared to 73.5% of those in quintile 5 (the richest). Therefore, not only are students from IDACI quintile 1 underrepresented in the top grades at the end of Key Stage 2, but their chances of progressing to GCSE grades 7 and above are also reduced. More support for students from IDACI quintile 1 is needed both during and after Key Stage 2 if they are to remain in the *excellence stream*. Progression rates are similar for students with different ethnicities, with the exception of Asian students who are 13-15 percentage points more likely to stay in the *excellence stream* during this period.

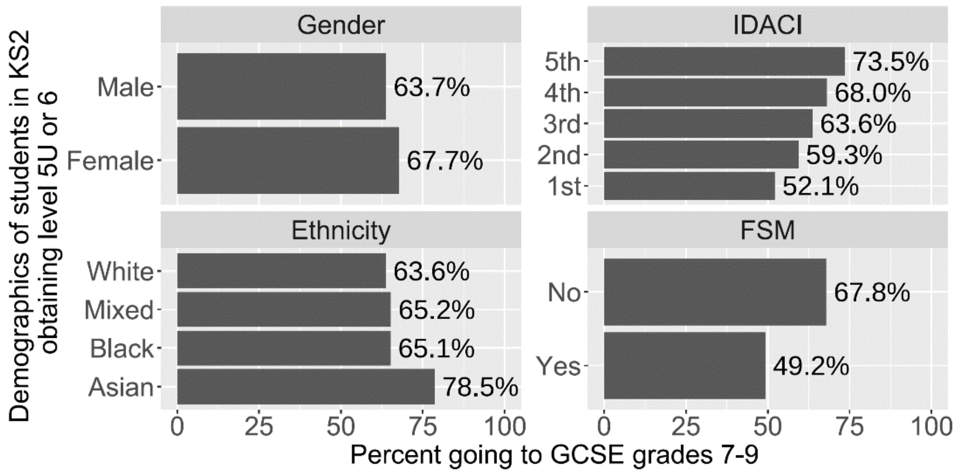

Figure 5: The percentage of students in Cohort 1 who achieved a good level 5 (i.e. the top half of this group) at Key Stage 2 who went on to achieve grades 7 to 9 at GCSE, broken down by gender, ethnicity, IDACI quintile and Free School Meal (FSM) eligibility.

Looking at the transition from GCSE to A level, Figure 6 shows the proportion of students in Cohort 1 achieving grades 7 to 9 at GCSE that progress to A level Mathematics. 61.3% of the high-achieving male students progress to A-Level compared to 41.7% of high-achieving female students, which leads to the A-Level cohort being male-dominated (see Figure 4a). Interestingly, Figure 6 also shows the progression rate for high-achieving students does not depend on IDACI quintile even though quintile 1 students are underrepresented in both GCSE grades 7-9 and A Level Mathematics (see Figure 4b). Finally, Figure 6 shows the progression rate for high-achieving white students is less than other ethnicities.

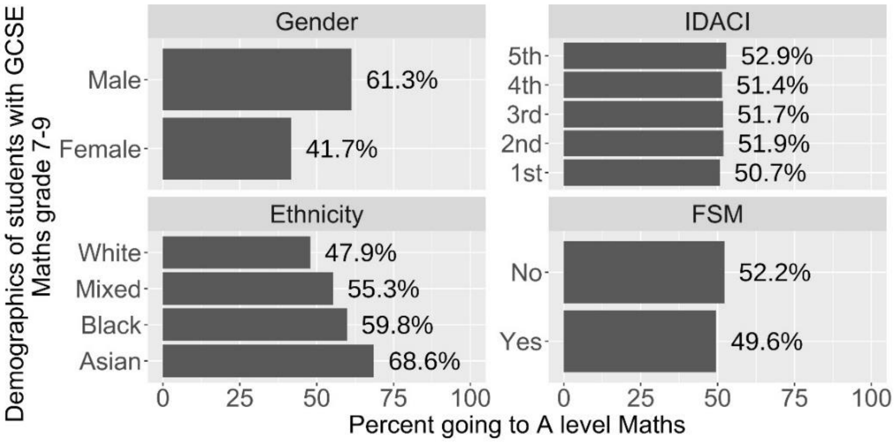

Figure 6: The percentage of students in Cohort 1 who achieved grades 7 to 9 at GCSE who went on to complete A level Mathematics, broken down by gender, ethnicity, IDACI quintile and Free School Meal (FSM) eligibility.

Figures 7 and 8 show the university course selected by students in Cohort 2 who achieve at least a grade A in A Level Mathematics. All the students in this group have the potential to study mathematics at university but Figure 7 shows only 17% of the male students and 13% of the female students choose to do so. After mathematics, the next most popular destination within this group of students in the *excellence stream* is medicine, which attracts 7% of male students and 12% of female students. Other popular subjects include physics, economics, mechanical engineering, computer science and chemistry, especially for male students, while female students are more diverse in their subject choice. Female students choose subjects in the life and social sciences more often than males, while male students are more likely to stay within mathematics or subjects closely allied to mathematics.

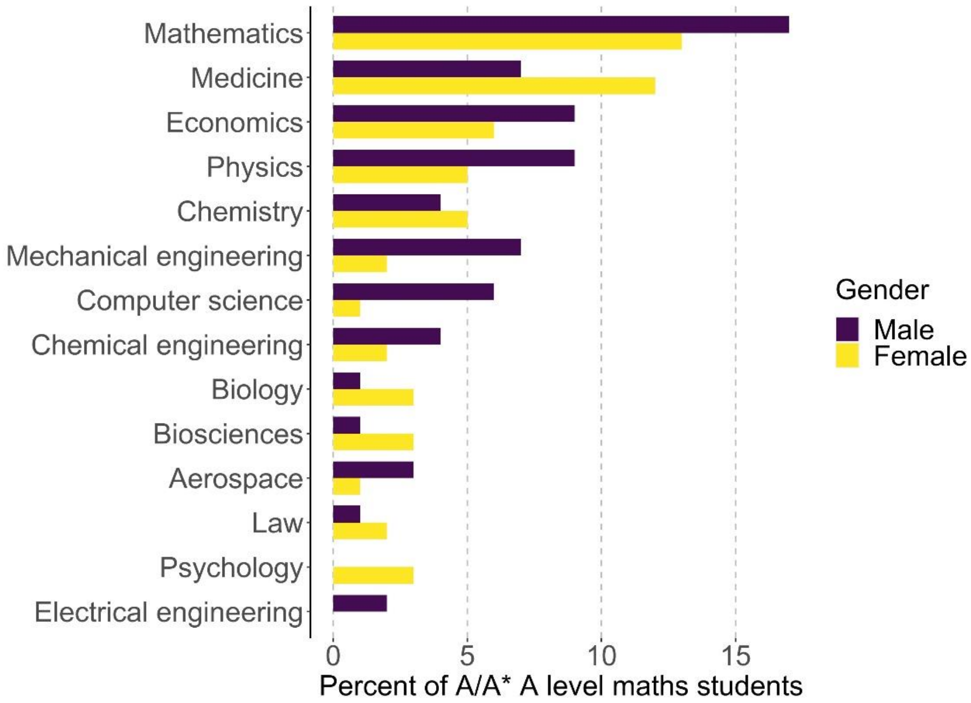

Figure 7: The degree subject choice of students in Cohort 2 who achieved grade A or A in A level Mathematics, shown as a percentage of male and female students. Only the 14 most popular subjects are shown.*

For the same subset of students, Figure 8 shows a strong relationship between subject choice and HESA's measure of socio-economic class (based on the occupation of the highest earning parent or guardian for students under the age of 21). For students from the lowest socio-economic classes (SEC) who achieve at least a grade A in A level Mathematics, 21% choose to study mathematics compared to 13% of those from the highest SEC. Overall, students from lower SECs are more likely to choose more vocational subjects such as engineering disciplines and subjects closely aligned to mathematics compared to students in the higher SECs. In contrast, students from higher SECs are more likely to branch out into medicine and economics and may have taken A level Mathematics to enhance their applications to these more selective courses. Additionally, it may reflect that students from lower SECs are more cautious and risk-averse in their degree subject choice, choosing courses perceived to have lower social and cultural barriers. Mathematics does well to keep students from lower SECs in the pipeline but, comparatively, loses students from higher SECs who may have a wider range of opportunities to choose from.

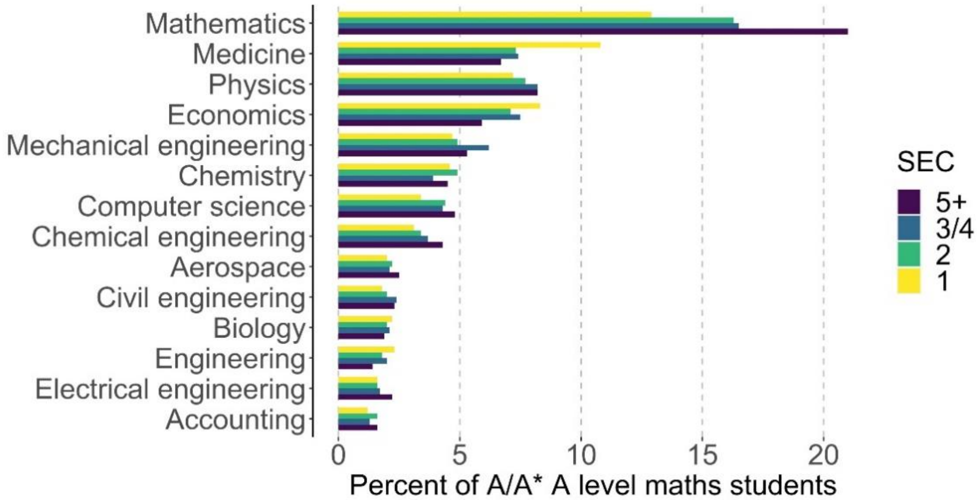

Figure 8: The degree subject choice of students in Cohort 2 who achieved grade A or A in A level Mathematics, shown as a percentage of students by HESA's measure of socio-economic class (SEC). Only the 14 most popular subjects are shown. SEC classes 1 and 2 are higher and lower, respectively, managerial and professional occupations; classes 3 and 4 are intermediate occupations and classes 5 and above are technical, semi-routine and routine occupations or long-term unemployed.*

Figure 9 shows that 82% of students in Cohort 2 who started a Mathematics degree successfully completed a Mathematics degree within 5 years (with the remaining 18% either transferring to a different subject or leaving with a lower or no award as shown in Figure 2). This completion rate is the same for male and female students. However, students of black, mixed and Asian ethnicities have a lower completion rate than their white counterparts, with the completion rate for black students being notably less than other ethnicities. When controlling for prior attainment, Asian students have particularly good progression rates from KS2 to GSCE and GCSE to A level (see Figures 5 and 6), but this effect is not observed at university level. Figure 9 also shows the completion rate is higher for more affluent students as measured by either IDACI quintile or HESA's socio-economic class measure based on parental occupation. When controlling for prior attainment, this means that despite students from lower socio-economic classes being more likely to start a Mathematics degree (see Figure 8), the inequality widens again in the Higher Education phase of the mathematics pipeline (c.f. Figure 4b). All of these points do not take account of any between-institution variation.

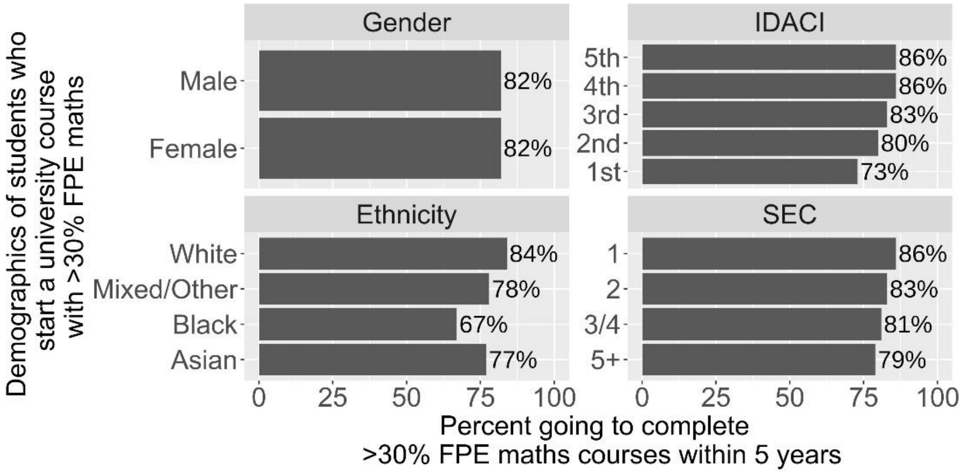

Figure 9: The completion rate for Mathematics degrees for students in Cohort 2, broken down by gender, ethnicity, IDACI quintile and HESA's measure of socio-economic class (SEC). The completion rate is the number of students who complete a Mathematics degree (where mathematics modules comprise at least 30%) as a percentage of those who started. SEC classes 1 and 2 are higher and lower, respectively, managerial and professional occupations; classes 3 and 4 are intermediate occupations and classes 5 and above are technical, semi-routine and routine occupations or long-term unemployed.

2.3 Summary

The flow of students through the mathematics pipeline, and in particular within the *excellence stream* is patterned by prior attainment and predicted by social characteristics. Many of these factors are well known in the literature (see section 3.1.2), though studies rarely look along the whole pipeline to see the cumulative effects, and points where trends start to appear or change. So, whilst it appears that the composition of the *excellence stream* is fairly consistently patterned along SES lines, this deteriorates from the end of Key Stage 2 to GCSE and is at its worst at A level. This strong filtering might help to explain why the comparatively few students left from IDACI quintile 1 are more likely to progress to a degree in mathematics than their more advantaged peers. It would be better to have the same rate of progress yet have many more of this group still in the *excellence stream* at age 16. Similarly, the gender gap in the *excellence stream* can be seen throughout the pipeline apart from at GCSE, though it must be recognised that female students' attainment at GCSE is generally better than males and this near 'equivalence' in grade 7-9 attainment for GCSE Mathematics is therefore only part of the story.

The gender gap can be seen most starkly in the transition to A level, but the analysis above also highlights a gendered difference in undergraduate choice patterns. Perhaps the mathematical and data sciences might reflect on the sorts of programmes that would be more attractive to those students (more often females) who, after outstanding attainment in A level Mathematics, opt instead for courses in the life or social sciences. On the other hand, it is reassuring that undergraduate mathematics study has relatively strong appeal for students from the most disadvantaged backgrounds. In the next section we draw on the extant literature in order to shed some light on the complex factors that influence these patterns in the national data.

3. Understanding patterns in the pipeline

In order to make sense of patterns of progression, attainment and participation discussed above, this section draws on two data sources: 1) a range of research and other literatures, and 2) stakeholder perspectives.

A review of literatures

The narrative review synthesised academic and grey literatures as well as statistical releases that present trends over time of relevance to the study. Given the large number of academic studies that have investigated characteristics of different stages of the pipeline, systematic reviews[33] and meta-analyses were prioritised. The review was initially organised into four educational stages 1) reception - primary (age 4-11); 2) secondary (11-16); 3) post-16 (16-18); and 4) university mathematics (18+) though some issues bridge these phases. The following four questions guided the review of literatures:

1. Who is engaging in this stage of the mathematics pipeline and how well?
2. What informs the mathematics that is taught during this stage of the pipeline?
3. What patterns of attainment exist during and at the end of this stage of the pipeline?
4. What key changes have occurred in this stage of the pipeline over the last 30 years and why?

Stakeholder interviews

The stakeholder interviews explored experts' perspectives on interventions that focused on one or more of the following: *engagement*, *progression*, *attainment* and *participation*. 'Expertise' was based on current or previous involvement with mathematics education and/or interventions. Interviewees represented four stakeholder categories, with some being in multiple categories:

- **system thinkers** who take a high-level view of the pipeline; they include those with decades of involvement in national developments in school or university mathematics education, or civil servants overseeing aspects of mathematics education in England;
- **educational leaders** who have institutional and student-level views of the pipeline and understand policy implementation and the challenges of educational change; they might lead large MATs, maths schools or university departments;
- **intervention facilitators** who design interventions for different stages of the pipeline; these vary in position (e.g. educational 'insiders' and external philanthropists) and their interventions can be of wholly different scales and funded in a variety of ways;
- **enrichers** who can often stand back from the pressures of modern education systems; they might focus on extra-curricular activities and often take a broader view of the pipeline, or none at all as they are focused on a narrow subset of the school population.

An initial group of 19 interviewees were identified. In addition, further conversations with several other stakeholders and interested parties enriched this dataset. The interview schedule was designed to explore interviewees' perspectives on the motivations for and effectiveness of interventions, awareness of other mathematics interventions and futures thinking for the mathematics pipeline. The interviews lasted up to an hour and were conducted by a member of the research team between April and June 2022, either online or face-to-face.

[33] Early in the project, we considered the pros and cons of conducting a systematic review ourselves but concluded that there was not one clearly defined theme on which to base such a review.

The review of literatures and analysis of the interviews generated important ideas and insights about influences on, and features of, the mathematics pipeline. These are organised below under five key themes: 1) social patterns of attainment and participation, 2) affect and attitude to mathematics, 3) curriculum and assessment, 4) stages and transitions, and 5) teachers and teaching.

3.1 Key themes

3.1.1 Social patterns of attainment and participation

Section 2 highlighted important variations in mathematical outcomes that are associated with social characteristics including SES, ethnicity and gender. These have been persistent concerns over decades and much reported in the literatures. Of the general attainment gap between disadvantaged students and those from more affluent backgrounds that has formed by age 16, 40% can already be accounted for during the pre-school years. A further 20% of the attainment gap has been shown to develop during the primary phase and another 40% during the secondary phase of education, culminating in a cumulative gap of up to nineteen months by age 16 (Hutchinson et al., 2016).

With a specific focus on the *excellence stream*, the 2022 provisional results for the end of Key Stage 2[34] suggest that the pandemic has had a negative impact on the disadvantage gap with just 12% of disadvantaged 11-year-olds achieving higher than the expected standard in mathematics compared with 27% of those not known to be disadvantaged. This suggests a restricted flow along the mathematics pipeline for socially disadvantaged students. Further evidence from the 2019 national assessment data[35,36,37,38] and in multiple large scale research studies shows that inequalities exist for both some ethnicity groups and girls at further stages along the mathematics pipeline (Elliot Major & Parsons, 2022; Sammons et al., 2012; Shaw et al., 2017). Geographical analysis of student outcomes for mathematics indicates that a larger proportion of the highest GCSE grades (7-9) are achieved in the more affluent south of England[39]. These inequalities impact on employability and earning potential, the opportunity to pursue further mathematical study and to contribute as an engaged citizen.

Prior attainment at GCSE is the strongest predictor of participation at A level. Once GCSE attainment is included in the predictive models, the effects of SES all but disappear. In effect, social class differences have already been ‘baked-in’ to GCSE outcomes by the time students choose A levels (Boylan et al., 2016; Noyes, 2009). This is important because it means social patterns of underachievement, formed at earlier points in the pipeline most likely contribute to students avoiding A level Mathematics or any other mathematical study beyond 16, for that matter (e.g. Mcmaster (2017); Noyes and Adkins (2017)).

Higher education is a driver of social mobility, with income gaps narrower amongst graduates than non-graduates. For mathematics, 42.5% of graduates who previously qualified for free school meals are in the top quintile of earners (Sutton Trust, 2021). Despite efforts by all universities to widen participation for underrepresented groups (BIS, 2016), the literatures show that in the higher education phase of the pipeline there are considerably fewer students with low SES compared to the population, or indeed, the higher education sector (Sutton Trust, 2021). Those who do enter higher education with low SES are

[34] https://explore-education-statistics.service.gov.uk/find-statistics/key-stage-2-attainment/2021-22
[35] https://www.gov.uk/government/statistics/early-years-foundation-stage-profile-results-2018-to-2019
[36] https://www.gov.uk/government/statistics/national-curriculum-assessments-key-stage-2-2019-revised
[37] https://www.gov.uk/government/collections/statistics-gcses-key-stage-4
[38] https://www.gov.uk/government/statistics/a-level-and-other-16-to-18-results-2018-to-2019-revised
[39] https://analytics.ofqual.gov.uk/apps/GCSE/County/

also less likely to complete their course and more likely to leave with a lower degree class (Crawford, 2014), with the gaps persisting into postgraduate study and employment (Sutton Trust, 2021).

National policies and interventions have sought to ameliorate these trends (Farquharson et al, 2022). The recent pandemic has, however, only widened the disadvantage gap (Rose et al., 2021) and this legacy is likely to exist for years to come. The challenge is to reach those students in most need whilst at the same time avoiding the singling out of particular groups that might lead to undesirable and marginalising side effects. One of the interviewees described how their intervention for primary and lower secondary students, was designed to address this:

> if you look at pedagogic approaches that are particularly effective for low prior attainment … or for high prior attainers, but who are less confident…or for any other group…almost always, the strategies that are good for targeting the needs of a specific group, benefit everybody. (Intervention facilitator)

Despite the attempts to address issues of inequality, the evidence points to a need for more efficient and/or different interventions that change social patterns of attainment at early stages of the mathematics pipeline and that benefit as many students as possible. This is particularly important for those students who have the potential to join the *excellence stream*.

3.1.2 Affect and attitudes to mathematics

Attitudes is a rather nebulous term but encompasses how students value, enjoy and take interest in mathematics. Developing and sustaining interest involves not only raising students' awareness of the usefulness of mathematics for employment and citizenship but also that it has cultural significance and is enjoyable in its own right. The latter is a challenge to some students' beliefs acquired from, amongst other places, their family milieu, and this is particularly so for students from disadvantaged backgrounds where negative attitudes to mathematics tend to be greater than for more affluent peers (Quaye & Pomeroy, 2022).

Positive attitudes to mathematics are important because they are associated with increased attainment (Richardson et al., 2020) and predict mathematical achievement in later years (Dowker et al., 2012; Evans & Field, 2020). In the early years, young children experience mathematical ideas through their natural curiosity about their environment. Sustaining this curiosity and interest for young learners can contribute to better mathematics outcomes (Shah et al., 2018). In international comparative tests (TIMSS[40] 2019) most Year 5 students reported liking mathematics, with significantly more boys than girls indicating that they *very much liked* the subject (Richardson et al., 2020). This echoes similar gender patterns at later points in the mathematics pipeline with male students appearing to have a greater preference for mathematics than female students. Attitudes to doing mathematics influence students' choices to study the subject beyond 16 (Sheldrake et al., 2015). Perceived difficulty, boredom and lack of relevance were reasons given by Year 10 students for not choosing the subject (Brown et al., 2008), whereas intrinsic motivation influenced Year 12 students' intentions to study mathematics at university. Hodgen, Küchemann et al. (2010) reported that the older secondary students got, the more negative their attitudes to mathematics became and that the decline was greater for girls than boys. We suspect that this has not changed over recent years.

The importance of sustaining positive attitudes towards mathematics cannot be underestimated, but nurturing these attitudes is complex, requiring considerable efforts from all those who influence the

[40] Trends in International Mathematics and Science Study https://timssandpirls.bc.edu/timss-landing.html

students. Done well, this can lead to the creation of communities of like-minded students, where mathematics is viewed positively. One interviewee referred to the importance of this type of community:

> It is a significant impact on [student] self, on who they are and who they want to be…the most common thing you'll find from our students is excitement around mathematics and a sense of finding their tribe. (Educational leader)

Several interviewees led interventions which aimed at improving students' attitudes to mathematics with the longer-term goal of encouraging post-compulsory study of mathematics:

> We're not just trying to get them through the exams, but to make them confident, engaged…to enjoy maths, to see it as something that's worthwhile and that is beneficial, whatever they end up doing in their future lives. Some of them will enjoy it and be engaged enough to consider mathematics as a career, maybe as a teacher, maybe as a researcher. (Enricher)

This sense of belonging has also been shown to motivate undergraduates to persevere with studying mathematics at university (Brown et al., 2005). Conversely, the sense of not belonging carries the risk of marginalisation and of perceiving the subject as irrelevant and dull. One of the interviewees indicated how their intervention considered this challenge through exposure to positive role models that were relatable to students:

> 'If you can see it, you can be it'…the idea of having a role model that uses maths which challenges those conscious and unconscious biases…around what the mathematician is, what a mathematician does, where they come from, what they look like…improves [students'] awareness of potential careers and…also has a benefit for the educator in the classroom as well. (Intervention facilitator)

The above evidence suggests that focusing on building and sustaining students' positive attitudes to mathematics and encouraging positive mathematical identities, well before the point where many students first leak from the pipeline, has the potential to improve levels of participation when mathematics becomes an optional course of study at age 16.

3.1.3 Curriculum and assessment

The mathematics that is learned, and how and when it is assessed, has a strong influence on students' experience in the mathematics pipeline. Since the introduction of the first statutory national curriculum in 1989[41], iterations have reflected the priorities of the government and ministers of the day, with slightly different balances in the broad purposes presented above (mathematics for employment, for its own sake and/or for citizenship).

Providing a common curriculum for all throughout the pipeline is problematic. It assumes homogeneity of students' capabilities, potentials, motivations and future interests, and that there is a single purpose for mathematics education. Several of our interviewees considered 'mathematics for its own sake' (in particular reasoning, problem solving and 'thinking like a mathematician') to be weak in the curriculum experience of many students. One interviewee commented on how the existing curriculum created opportunities for their intervention to offer older, high-attaining students experiences of familiar topics such as geometry in greater depth, or topics not included in the school curriculum such as combinatorics, without interfering with the students' school experiences:

[41] https://www.legislation.gov.uk/ukpga/1988/40

> So, it means that when we are providing support to the most able students, we're not actually getting in the way. We're never finding ourselves irritating school teachers…we're teaching them something that they wanted to teach them. (Enricher)

The current national curriculum was launched for first teaching in September 2014 with the laudable aspiration that the programmes of study “embody high expectations and are designed to raise standards for children aged 5-16, especially the poorest”[42]. This reflects a belief that it is possible to simultaneously achieve a curriculum ‘for all’ and ‘for excellence’, enabling students to excel irrespective of background. It is clear, however, from the above that student outcomes are not equitable and far from homogenous. Some progress through the pipeline at a reduced rate, others are lost from the pipeline altogether, whilst others race along in the *excellence stream*, all of which creates tensions between the rhetoric of ‘mathematics for all’ and that of ‘mathematics for excellence’.

Adapting the curriculum for pupils in the *excellence stream* is one option. This is attempted, to a degree, in the Key Stage 4 curriculum where the content is expanded in some mathematical topics for higher attaining students. The GCSE foundation and higher tier papers also vary in content weighting. In the foundation tier, papers include questions with a greater focus on number and using and applying standard techniques, whereas the higher tier questions involve a greater emphasis on algebra and geometry along with mathematical reasoning and non-routine problem solving.

One of the challenges in curriculum design is deciding what mathematics to teach when, and how interest in further study might be stimulated by exposure to some topics in greater depth and earlier. One of the interviewees commented on how they were thinking about this:

> … you can't just do it all up at Level 3 because it's what happens to them before that really has an impact. So, we started to develop a series of courses around extension and enrichment within higher tier GCSE to start getting teachers to think about the continuation of certain topics that they might study at GCSE through to A level Maths and Further Maths. (Intervention facilitator)

In England, students’ attainment in mathematics is assessed in statutory tests or examinations at the ages of 5 (Early Learning Goals), 7 (end of Key Stage 1 tests), 11 (end of Key Stage 2 tests) and 16 (GCSE) and for those opting to study advanced mathematics qualifications beyond 16 (e.g. Core Maths, A level Mathematics). In addition, from 2021, 9-year-olds (Year 4) have had to demonstrate competence in the multiplication tables check (MTC)[43]. This pattern of national testing carries the risk of distorting students’ experiences of mathematics with an over-emphasis on individual performance on timed written tests. Whilst GCSE outcomes are important for an individual student’s future, collective student outcomes at several of these assessment points are also used to hold schools to account. The high-stakes nature of the assessment is both individual and institutional.

Although politicians argue that high-stakes tests drive up standards, such assessments have been shown to obey Cambell’s Law[44] leading to negative effects for students, such as experiencing a narrowing of the curriculum (Berliner, 2011). In mathematics, this can mean students spend more time on routine and practice tasks to develop fluency in solving ‘typical’ test/examination questions. Such practices emphasise procedural knowledge in favour of problem solving and reasoning capability that can contribute to conceptual understanding (Kilpatrick et al., 2001). Whilst the former may achieve the

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[42] https://www.gov.uk/government/news/new-curriculum-will-make-education-system-envy-of-the-world
[43] https://www.gov.uk/government/collections/multiplication-tables-check
[44] The more any quantitative social indicator is used for social decision-making, the more subject it will be to corruption pressures and the more apt it will be to distort and corrupt the social processes it is intended to monitor (Cambell,1979).

broad purpose of mathematics for employment and application, it is less likely to produce the kinds of mathematical thinkers that admissions tutors in universities are looking for. Nunes (2009) showed that primary students from more affluent backgrounds were better at mathematical reasoning and argued that "improving reasoning through instruction could make an important contribution to reducing SES inequalities" (p. 5). Narrowing the mathematics curriculum as described above compounds the disadvantage gap and access to the *excellence stream*. This was a priority intervention area for one of the interviewee's who described this as

> an unnecessary perceived tension between teaching for understanding and teaching for success in high stakes assessment and a perceived but unnecessary tension between being able to do maths and being able to understand maths [which] are not present in some of the higher performing jurisdictions (Intervention facilitator)

Some recent reforms have sought to redress this imbalance of mathematical purposes, particularly for those students in the *excellence stream*. The new GCSE Mathematics introduced for first assessment in 2017 was designed to increase the level of challenge[45] and the reforms to A level Mathematics two years later established a common curriculum (including statistics and mechanics) and a new emphasis on problem solving and modelling. Whether or not these have led to substantial change is a moot point.

Curriculum and assessment in the mathematics pipeline becomes less constrained at the transition to university; there is a wide variety of undergraduate mathematics curricula on offer at different HE institutions (QAA, 2019). Common elements include linear algebra and calculus but different branches of mathematics such as analysis, number theory, mathematical modelling, scientific computation, probability, statistics and mathematical physics, are covered to different depths. In contrast to the high-stakes paper-based assessment at GCSE and A level, there is also greater emphasis on computing, written reporting and oral presentation types of assessments that sometimes involve groupwork.

Different governments have repeatedly claimed that curriculum reform is a mechanism for driving down social inequalities yet, despite this, successive iterations of the national curriculum for schools in England have not achieved such. Some even argue that the inequalities are the product of such endeavours (Jackson, 2022). Those without political influence have limited agency to effect any changes on the national curriculum and its assessment. However, localised interventions can expand the offer and contribute to enriching students' experience of mathematics, thereby retaining students in, or drawing them into, the *excellence stream*.

3.1.4 Stages and transitions

The mathematics pipeline in England is divided into several stages with transitions between some of these typically involving changes of institution. Much is known about how well students achieve in mathematics at each of these stages from national assessment data as well as analyses of student outcomes in international comparative tests (e.g. TIMSS[40], PISA[46]). Whilst there is always room for improvement at any stage of education, the most recent TIMSS data (Richardson et al., 2020) indicate that, overall, students in primary school in England generally achieve well and make good progress in mathematics compared to other international jurisdictions and other stages of the pipeline in England, even when factors such as disadvantage are considered.

The transition from primary to secondary school is an educational 'rite of passage' bringing new experiences, some of them daunting and others exciting. Multiple studies show that the first important

[45] https://www.gov.uk/government/speeches/reformed-gcses-in-english-and-mathematics
[46] Programme for International Student Assessment https://www.oecd.org/pisa/

damage to the mathematics pipeline follows primary-secondary school transfer where there is a dip in mathematical attainment and attitudes to mathematics (Brown et al., 2003; Jindal-Snape et al., 2020; Kaur et al., 2022). This no doubt also impacts on the *excellence stream*. Ofsted (2015) coined Key Stage 3 the ‘wasted years’, observing slow progress and a lack of challenge for the ‘most able’ pupils in mathematics (p. 5). The transition *per se* cannot fully account for this ‘dip’ as there are other factors at play such as students’ attitudes to mathematics, student-teacher or peer relationships and the wider school environment (Evans et al., 2018; Evans & Field, 2020; Jindal-Snape et al., 2020).

Several interviewees referred to this stage of the pipeline as a much-needed area for attention with potential for longer term benefits. One interviewee concerned with post-16 mathematics explained that school leaders needed encouragement to think about the mathematics pipeline journey on offer in their schools from year 7 onwards to increase later participation,

> [They] should be thinking, from Year 7 that all these students are going to choose A-level, or will need to study something post-16 that has got maths in it,…so what are they doing with them at various stages through KS3 and KS4? …In the future, schools would really think carefully about a systematic program. And that might involve us. Or it might involve other people. (Intervention facilitator)

Many studies that have explored patterns of post-16 mathematics (ACME, 2012; Hillman, 2014; Noyes & Adkins, 2017). These identify several general patterns, for example, 1) students with higher GCSE mathematics grades are more likely to choose A level Mathematics and succeed; 2) for similar prior attainment at GCSE, girls are less likely to choose A level Mathematics than boys and 3) for similar GCSE grades, disadvantaged students have similar likelihood of continuing to A level Mathematics, however students from low-income backgrounds are much less likely to get the top GCSE grades.

A level participation rates also vary considerably across regions in England, representing a clear North-South divide. In 2015-16, the local authorities with the lowest participation rates were all in the north of England with fewer than 20% of the students achieving A*-C at GCSE going on to study A level Mathematics (Smith, 2017). As mentioned above, the highest GCSE Mathematics grades in 2022 were awarded in ‘the South’[47], suggesting these geographical variations are resistant to change. This is important because the supply of A level candidates is dependent on GCSE grades so these regional differences persist into post-16 mathematics participation, although it is noteworthy that the highest grades (A*/A) for A level Further Mathematics are more evenly distributed nationally[48]. Other studies have suggested that students believe pursuing mathematics beyond 16 is only for an ‘elite’ (Nardi & Steward, 2003) or ‘clever core’ (Matthews & Pepper, 2007) and females still tend to think of STEM careers as being ‘male dominated’ (Cassidy et al., 2018).

Approximately 88,000 students (33,000 female) sat A level Mathematics in 2022. 47.3%[36] of those students achieved a grade A or A* and of around 14,000 (4,000 female) who also took A level Further Mathematics, 64.7%[36] achieved the highest two grades. What is notable is the low proportion of female students taking A level Mathematics. These high attaining A level Mathematics students represent those capable of continuing with mathematics to undergraduate level. However, just under 7,000[15] students from England apply for and are offered places on mathematical sciences courses each year. Approximately 4% of mathematics undergraduates are eligible for Free School Meals at age 16 (Key Stage 4). This compares to 6% for all undergraduates and 12.5% for the population (Sutton Trust,

[47] https://analytics.ofqual.gov.uk/apps/GCSE/County/
[48] https://analytics.ofqual.gov.uk/apps/Alevel/County/

2021). Supporting lower income students through this transition from school to university, so that they successfully complete their studies is an important focus for preventing pipeline leaks at this stage of the *excellence stream*. Mathematics undergraduates leave mathematics courses before completion for several reasons e.g. diminishing success and enjoyment of the subject, isolation due to living at home, limited study skills (Brown et al., 2005), or balancing studying with paid employment. One of the interviewees described a new programme designed to address some of these factors:

> it's sort of a six-month period…at the beginning…they will meet [a tutor] at key points…where we think there might be moments of doubt…So after you get your first assignment back, you're used to always getting A grades. Suddenly you've got a thing called a third and you didn't know what that [is]. And you think, "Oh my gosh, I'm gonna have to leave the university. It's not for me"…Making sure that there's a touch point then (Intervention facilitator)

A survey of undergraduate applicants in 2015 found that "the earlier young people understand about the opportunities available through HE, the more likely they are to be motivated to apply" (UCAS, 2016, p. 7). Students from disadvantaged backgrounds are the most likely to have adopted that goal much later in their education. One interviewee described the importance of their intervention as extended influence on working with primary students, particularly from disadvantaged backgrounds,

> It's long term because we think that being a kind of stable presence in young people's lives is important …a lot of what we do is based on the relationships that we form with young people …all of the people that work in centres have gone to uni so this gives young people who may be the first in their family to go to university…a person that they know who's been…who can talk positively about it (intervention facilitator)

The final stage in the mathematics pipeline is postgraduate study. It is increasingly essential that students can access Masters programmes in order to adequately prepare for PhD research programmes (Dixon & Vittle, 2014). The provision of undergraduate Master's (Mmath) programmes, with their capped tuition fees and government backed student loans, offers one funding route. For standalone Masters programmes, the introduction of loans in 2015/6 narrowed the gap in participation between different socio-economic classes (Wakeling & Mateos-Gonzalez, 2021). However, the same study showed that after controlling for prior attainment and institution notable disparities remain with students from more affluent backgrounds more likely to progress. One interviewee highlighted negative student attitudes to taking on further debt,

> I think [the funding situation has] improved, but a lot of the students I talk to say well, I'm already sort of £40,000 in debt. I'm not sure I want to get another ten grand in debt. (Educational leader)

At PhD level, the main constraint on the pipeline is funding. The Bond Review (2017) called for a tripling in public-sector funding for mathematical research and the creation of at least 100 additional PhD places per year for training mathematical scientists.

The above evidence suggests that there are three points that require attention to retain more students beyond age 16. Firstly, following transition to secondary school; secondly, prior to transition to A level study; and thirdly, navigating the transition to mathematics at university amidst a wide variety of competing opportunities and, for some, economic and cultural barriers. An overarching factor associated with these transition points is the way that studying mathematics for longer is signalled to students. There are two types of signalling required: 1) enabling all students to appreciate mathematics as purposeful – for employment, for its own sake and for citizenship, and 2) enabling students from underrepresented groups (e.g. females, low SES) to identify with those purposes. Teachers have important roles to play here but need support as the future mathematical needs of students are highly diverse.

3.1.5 Teachers and teaching

Teachers are key enablers of students' progression in mathematics. There are approximately one quarter of a million (full time equivalent) teachers teaching mathematics in state-funded nursery, primary and secondary schools[49]; that is over half of the teacher population in England. There is wide variation in qualifications held by these teachers and there are multiple routes into the profession. The government has pledged[18] to create a 'world-class' initial teacher education (ITE) market coordinated by a new *Institute of Teaching* and a commitment to providing a 'golden thread of teacher development' at every stage of a teacher's career[50]. Primary teachers are required to have achieved at least grade 4 (formerly, grade C or above) in GCSE Mathematics to join the profession; in the majority of cases, this is their highest mathematics qualification (DCSF, 2008). In contrast, approximately 44% of secondary teachers of mathematics hold a mathematical degree[51] (Allen & Sims, 2018).

The supply of mathematics teachers at secondary level remains an area of concern for government (Long & Danechi, 2021). In 2020-21 84% of the target enrolment to secondary mathematics ITE places was reached, continuing the trend of under-recruitment (Worth & Van den Brande, 2019). Recruiting below target is not the only contributory factor to the teacher supply challenge. Secondary teachers leaving the profession are increasing year on year, with an above average number of mathematics teachers leaving (Worth & Van den Brande, 2019).

Teacher shortages are managed locally by schools. Senior leaders deploy mathematics teachers in secondary schools by placing more experienced and better qualified teachers where the stakes are higher, i.e. with GCSE and A Level students (Allen & Sims, 2018) and as a consequence "the shortage…is being felt most keenly at Key Stage 3" (p. 5). Furthermore, in schools located in areas of deprivation, there are often insufficient teachers with appropriate professional experience and qualifications to teach classes preparing for high-stakes assessment (ibid.). One interviewee referred to how a school in these circumstances tried to address this challenge by producing lesson scripts for 'non-specialist' teachers. For example, these might include,

> questions you will ask and…a flow chart of the pupils' [responses], this is your response…, but they were very clear that was necessary because of the number of non-specialist staff they had…They identified they couldn't possibly flow chart every possible response,…when they observed these staff…that there [were] missed opportunities because those colleagues didn't have that breadth of mathematical knowledge. (System thinker)

This paucity of suitably qualified teachers can also impact on the (quality of) provision of A level Mathematics or Further Mathematics (Hillman, 2014).

Teachers' mathematical subject knowledge was frequently mentioned by interviewees as being an important aspect of teacher quality. Studies on the mathematics teacher workforce have highlighted a lack of a clear agreement about what it means to be a 'specialist teacher' of mathematics (Allen & Sims, 2018; The Royal Society, 2007). Teachers have access to subject specific professional development through centrally funded NCETM Maths Hubs activities or from other commercial, independent, and

[49] https://explore-education-statistics.service.gov.uk/find-statistics/school-workforce-in-england;
[50] https://www.gov.uk/government/publications/opportunity-for-all-strong-schools-with-great-teachers-for-your-child
[51] Allen and Sims used data from 2016.

local providers. In primary, for instance, NCETM has produced just over 1000[52] Mastery Specialist Teachers across the country who have undergone continuing professional development in seven cohorts since 2015. Participating in high quality professional development also has the potential to increase the likelihood of teachers staying in teaching for longer (Worth & Van den Brande, 2019).

Teachers' subject knowledge may go some way to improving the learning experiences for students, but teachers' own connection with mathematics can also influence how they teach mathematics. One interviewee reflected on this in the context of a year 5 teacher they had met who had achieved Grade C on the fourth attempt,

> …it's all action and energy and passion when she's talking about geography or English and maths is very much stifled and constrained, that's inevitably going to influence the experience pupils get. (System thinker)

This same interviewee pointed to the importance of teachers immersing themselves in some mathematics as a part of professional identity, but this might be problematic for those like the year 5 teacher:

> actually there's no shortcut…it will always be, to some extent, a sticking plaster as long as we're not getting people who have…that love for maths throughout their entire career that they can share with pupils. (System thinker)

There are some teaching practices highlighted in the literatures that can influence flow along the mathematics pipeline. For instance, teachers who believe that all students can make good progress tend to have a more positive impact on student outcomes (Shaw et al., 2017). Conversely, low expectations of disadvantaged learners can have a negative effect (Hinnant et al., 2009) and this has been shown to be associated with how teachers group students. Grouping practices based on prior-attainment to create 'homogenous' classes (sets) have been shown to have a negative impact on students with low prior attainment and disadvantaged intakes at both primary and secondary (Bradbury & Roberts-Holmes, 2017; Francis et al., 2017; Solomon, 2007; Taylor et al., 2022). Despite this evidence, such practices are prevalent.

Effective teachers enhance students' mathematical engagement, progress and attainment. They have a pivotal role in signalling the various purposes for mathematics in their daily work with students. A particular challenge in schools in England is having sufficient numbers of appropriately 'qualified' teachers.

3.2 Summary

Three features of the mathematics pipeline are pertinent and point to areas for future interventions that could improve efficiencies in the mathematics pipeline and, in particular, the retention of more students from diverse backgrounds in the *excellence stream*. Firstly, students growing up in disadvantaged families are less likely to remain in the *excellence stream* than their more affluent peers. These students typically experience greater viscosity in the pipeline for an amalgam of reasons. Secondly, the fact that female students are more likely to drift from the *excellence stream* is a persistent concern, the earliest signs of which emerge at primary school where boys are more likely to like mathematics than girls. Thirdly, attainment is associated with affect and attitudes, though not necessarily in a causal relationship. Improving all students' attitudes to mathematics has the potential to increase general

[52] Personal communication with Debbie Morgan, Director for Primary, NCETM 30-08-2022.

interest in the subject. Finally, patterns of attainment and attitudes change along the pipeline, though these changes are most notable after transition to secondary school, prior to choosing A levels for study post 16, and in the move to university. These three sections of the pipeline arguably require the greatest attention if the *excellence stream* is to be increased and diversified.

The above features of the pipeline are influenced by curriculum and assessment, and by teachers and teaching practices. What teachers choose to prioritise in their teaching is influenced by what is assessed, which in turn skews and limits students' mathematical experiences. Some interventions create opportunities to expand on, and enrich, students' experiences of the curriculum. Communicating the different values, purposes and future applications of mathematics to students such that it becomes a subject worthy of further study is also important. Better signalling might therefore be another focus for interventions. In the next section we discuss how current interventions and initiatives act as enablers of, or mitigate barriers to, progression in the mathematics pipeline.

4. Interventions in the mathematics pipeline

Much of this report hitherto has been concerned with system-level patterns of student progress, attainment and participation, and of the various insights that can be gleaned from the research and from stakeholders. Some of those stakeholders were what was termed *intervention facilitators,* and so the landscape of mathematics pipeline interventions[53] is now considered.

This section is not concerned with the main policy apparatus of government and its Department for Education, for example national curriculum, qualifications, Ofsted, etc., critically important though they are, but rather with the multiplicity of interventions across the educational landscape in England. These can be targeted specifically at mathematics learning, or on supporting particular groups of learners, who may or may not be in the mathematics *excellence stream*. Government-supported initiatives that are either time-limited or not widespread, and which get implemented through various interdependent organisations and networks, are also included here.

The mathematics intervention and initiative landscape is shaped by different stakeholders, operating on scales ranging from a single institution through to national level, and with motivations that are sometimes complementary and sometimes competing. Furthermore, the landscape looks different at primary, secondary and tertiary phases, and is evolving over time. Indeed, the project team was surprised to discover the sheer range of such interventions, their overlaps and gaps, ambitions and limitations. It seems clear that there is very little orchestration of such interventions so that similar students can benefit from them irrespective of where they live or study. For any education leader, the array of options might seem confusing or even bewildering.

This section will outline types of interventions by considering the providers, their motivations, the scale and organisation of delivery, timescale and intensity. This leads to the discussion of a typology for interventions across the mathematics pipeline that has been developed to enable a more systemic and critical analysis of individual interventions and the intervention landscape as a whole.

4.1 Some features of interventions

4.1.1 Stakeholders and motivations

Aside from the government, other important stakeholders in the intervention landscape are educators who are working to develop, research and share good practice, and philanthropic individuals and organisations that are aiming to address perceived deficiencies in the system.

Stakeholders' motivations vary with some approaches enhancing the mathematical ability of individuals in preparation for everyday life, others improving the long-term supply of mathematicians for industry, and others promoting mathematics because they are passionate about the subject itself. For example, the Every Child Counts[54] programme of interventions to support young children who are struggling with mathematics has a different motivation to the UKMT[55] whose mathematics challenges are aimed, in one sense, at selecting an elite UK team of older students for the International Mathematical Olympiad. In between are other interventions such as the AMSP[56] with its extensive remit, and nested national

[53] At this point we use 'interventions' in a rather general way to include anything beyond the business-as-usual activity of mathematics teaching. They might vary in scale, scope, objectives, funding source, etc. We define and discuss a framework for making sense of this intervention landscape later in this Section.

[54] https://everychildcounts.edgehill.ac.uk/

[55] UK Mathematics Trust https://www.ukmt.org.uk/

[56] Advanced Mathematics Support Programme https://amsp.org.uk/

structure, to increase participation and provision in post-16 mathematics education for the benefit of students considering a wide range of university courses and careers.

Sometimes motivations for a particular intervention may need to be re-shaped for it to complement other features of the mathematics education landscape. For instance, one interviewee explained how they took account of the pressures schools were under in terms of exam results and inspection.

> we have to find a way to work with that and…make sure that what we offer is something that schools and teachers want to use. (Enricher)

Another difference between interventions is the nature of their primary aims. Some, such as the Brilliant Club[57], aim to increase engagement through enhancing enthusiasm and interest while others, such as the National Tutoring Programme, are motivated by increasing attainment. Furthermore, some, such as Maths4Girls[58] are aimed at increasing participation in the later stages of the mathematics pipeline while others, such as Mathematics Support Centres in universities, are more focussed on progression within a student's current phase. It should be noted that most interventions have multiple complementary aims in terms of increasing engagement, progression, attainment and participation but may prioritise these differently.

4.1.2 Scale and organisation

Government curriculum supported initiatives, for example 'teaching for mastery', typically achieve national coverage but many other interventions, in particular those relying on philanthropic support, have limited budgets and are not so easily scaled. Interventions which are rolled out through the provision of teaching resources or professional development for teachers have potentially larger reach than those which rely on voluntary learner participation or one-to-one contact with learners. For example, Imperial College's mentoring programme[59] for A level students is limited to 100 participants per year due to the limited supply of undergraduate mentors, whereas their online A level enrichment courses are not capped.

As well as size, interventions differ in their geographical organisation. For example, the creation of maths schools is a central government initiative but has resulted in a handful of schools in different parts of the country operating largely independently from one other. On the other hand, Into University[60] which aims to increase university participation amongst under-represented groups, works through a network of 35 centres around the country but is focussed on working in particular neighbourhoods rather than working directly with schools or universities. Other interventions, such as the Stoke-on-Trent Mathematics Excellence Partnership[61], are collaborations between schools, the local university and other organisations. Sometimes geographical and organisational structures don't align. For example, multi-academy trusts may span schools in different parts of the country and so not align well with with AMSP who operate a model of supporting schools by Local Authority area.

4.1.3 Timescales and intensity

Some interventions have a long-term focus, where the benefits of the programme may only be fully realised several years after engagement. Many teaching strategies or careers resources, such as the Maths Careers website[62], would fall into this category. Other interventions, such as the use of

[57] https://thebrilliantclub.org/
[58] https://m4g.founders4schools.org.uk/
[59] https://www.imperial.ac.uk/be-inspired/schools-outreach/secondary-schools/mentoring-and-tutoring/maths-online-programme/
[60] https://intouniversity.org/
[61] https://sites.google.com/view/stokemaths/home
[62] https://www.mathscareers.org.uk/

contextual offers in university admissions procedures, have a much shorter-term focus and may only be relevant to a single phase or transition within the pipeline. Similarly, the intensity of the learner's experience of the intervention can vary widely. For example, foundation year programmes and university outreach programmes are both aimed at increasing and enabling participation in higher education, but the learner is fully immersed in the former whereas the latter may only be experienced intermittently at special events.

4.2 A typology of mathematics pipeline interventions

In order to understand the difference between interventions, a framework with eight dimensions is proposed below. The framework[63] is designed to aid intervention funders, designers and facilitators to clarify the purpose and scope of their intervention and also, importantly, highlight the limits for an intervention. The framework could also be utilised by system leaders and funders to survey existing interventions and government initiatives to identify any duplication or gaps. There is not sufficient published evidence to say which types of interventions have the greatest impact, and different issues may require different types of intervention. An intervention may span more than one category within a dimension but may prioritise one category over another (indicated in brackets in Table 1). The typology could no doubt be adapted further but its real value is not in being definitive but rather to raise questions and encourage scrutiny of an intervention and of the mathematics pipeline as a whole.

Dimension	**Categories**
Aim (1 or more) What are the intervention's main aims?	Engagement, Progression, Attainment, Participation
Phase (1 or more) How old are the participants?	Reception/Primary, Secondary, Post-16, Undergraduate, Postgraduate
Attainment (select 1) What is the general mathematical ability of targeted participants?	High-attainers, Low-attainers, Universal
Demographics (1 or more) What are the demographics of the participants?	Income, Gender, Ethnicity, Universal
Coverage (select 1) Where are the participants?	National, Local, Site
Concentration (select 1) What proportion of target audience participate?	Most (50%+), Some (10-50%), Few (<10%)
Dosage (1 in each) i) Over what period is the intervention? ii) How frequent is the intervention?	Short-term, Medium-term, Long-term Continuous, Intermittent
Model for delivery (select 1) How is the intervention facilitated?	Teacher PD, Teacher-directed, Self-directed, Other-directed

Table 1: Simplified typology of mathematics interventions to assess purpose and scope of potential and existing interventions (see Appendices for an expanded version)

The *aim* dimension is arguably the most important. Most interventions include multiple aims, but it is advisable to identify the principal aim as well as those aims of secondary importance. For example, an

[63] See Appendix 8.2 for a more detailed version of the framework.

intervention's main aim might be to increase mathematical attainment (e.g. at age 16), which will in turn unlock the opportunity for increased participation in post-16 mathematics.

The *phase*, *attainment* and *demographics* dimensions identify an intervention's target group. Together with *coverage* some idea of the population of eligible students can be discerned. *Coverage* captures something of the scale of an intervention based on geographical reach rather than the number of students because the number of institutions involved will have an impact on the nature of the intervention. 'Local' covers a range of mid-level scales and might include Opportunity Areas (OA) or the new (Priority) Educational Investment Areas (EIA/PEIA)[64], or be a medium/large MAT.

The number of students involved is approximated by combining the above dimensions with an estimate of the *concentration* of an intervention. In essence, concentration considers the extent to which an intervention reaches all of the eligible participants, although absolute numbers of students is unhelpful given the different scales involved.

Dosage, as in medicine, combines notions of the duration and frequency of the intervention. The framework does not specify a total number of hours since an intensive week-long intervention will have a similar number of hours as a one hour per week intervention spread over a year. Likewise, an intervention which comprises a regular commitment differs from a one-off intervention.

Lastly, the typology identifies the intervention *model for delivery.* This includes a place for interventions focused primarily on teacher professional development as an indirect route to achieving one or more of the four aims. It also includes initiatives that are channelled through educational institutions (*teacher-directed*) or outside of them (*other-directed*), and those interventions that are accessed by individuals (*self-directed*).

Carefully considering an intervention using the typology might enable a funder to raise important questions about its workability and potential impact, or an outsider to question claims about the scope and impact of interventions. For example, an intervention might be implemented nationally with low concentration and end up missing large numbers of those eligible. Alternatively, it might be implemented 'locally' in ways that ensure higher concentration. If the intervention cost is the same, which of these is preferable? This in turn relates to broader goals of the intervention designer, for example to evaluate it robustly and be able to scale out.

4.2.1 Evidence of intervention impact

Most intervention facilitators are able to provide some quantitative or qualitative evidence that their intervention is successful against their chosen metrics but there are multiple challenges associated with robustly evaluating interventions in the mathematics pipeline. Randomised controlled trials are often not possible due to ethical or practical considerations, and most interventions involve voluntary participation on the part of teacher or learner and therefore there is selection bias as well as random variation that needs to be accounted for. Some stakeholders aim to evaluate their interventions by comparing the participating cohort against a benchmark cohort with similar background characteristics. Some have commissioned independent evaluations to evidence impact and support the case for further funding. For example, Every Child Counts (1stClass@Number) was evaluated by the University of Oxford in 2018[65] and the Brilliant Club's Scholars Programme by the Universities and Colleges

[64] Further information on EIAs and Priority EIAs can be found at https://www.gov.uk/government/publications/education-investment-areas-selection-methodology.
[65] https://educationendowmentfoundation.org.uk/public/files/1stClass@Number_evaluation_report.pdf

Admissions Service (UCAS)[66]. Outreach interventions provided by universities are less likely to be independently evaluated, with HEFCE noting in 2015 that “relatively few of the interventions that have been initiated have been evaluated systematically” (Mountford-Zimdars et al., 2015, p. 93) and therefore their impact on the mathematics pipeline is even less clear.

A fundamental sector-wide problem in evaluating interventions is the difficulty of monitoring which learners are exposed to which interventions and tracking their progress for sufficient time to evaluate the long-term impact. For example, a primary school intervention may use end of Key Stage 2 results to measure an intervention’s effectiveness and a university outreach programme may use surveys to measure whether it raises aspirations, but these are only proxies for whether they increase participation and attainment in degree programmes. While linked data from the Department for Education and the Higher Education Statistics Agency have enabled us to track an individual’s progress through Key Stages at school and university, these data are not linked to any interventions that an individual might have experienced along the way. Establishing a better evidence base, perhaps including new, long-term evaluations of mathematics interventions would be valuable though challenging.

4.2.2 Some gaps identified in the intervention landscape

Many interventions and government initiatives are available nationally but their effectiveness may not be uniform due to local variations, such as the shortage of specialist mathematics teachers which is felt more acutely in some regions. Where local partnerships exist, for example universities and maths schools providing enrichment and resources to schools in their community, there is a risk that they focus narrowly on future enrolment rather than increasing the number of students staying in the *excellence* pipeline. Furthermore, maths schools, Russell Group universities and industries that could provide mathematician role models are not uniformly distributed across the country. For example, a child from an underrepresented group is more likely to attend university if they live in London than some more rural or coastal areas. The Stoke-on-Trent Mathematics Excellence Partnership is an unusual example of holistic area-based cooperation addressing local challenges for the benefit of all.

School based interventions are generally aimed at raising engagement, progression and attainment which should have long term benefits. However, at GCSE, post-16 and university level, interventions tend to be more short-term in their focus such as helping a student to make the transition to the next phase of education. There are very few interventions which provide continuity of support across institutions. One exception is the Brilliant Club’s new Join the Dots[67] initiative which seeks to join up the support received by a post-16 student from her teachers with that from mentors and coaches in their chosen university to make their transition to university smoother. For mathematics, MEI’s Integral resources for reviewing Year 13 topics also help in the early university phase. However, even in these two cases the support tails off six months into the university course and requires the student to quickly establish new support structures.

A lot of the education sector’s focus, in particular post-16, is on avoiding attrition in the pipeline but more attention could be paid to creating routes for individuals to get back into the pipeline. For example, there are conversion courses in data science[68] with government-funded scholarships and industry-backed placements. Such things do not exist for mathematics. There could also be more diversity in

[66] https://thebrilliantclub.org/news/ucas-evaluation-once-again-demonstrates-impact-of-the-brilliant-club-on-progression-to-highly-selective-universities/
[67] https://thebrilliantclub.org/join-the-dots/
[68] https://www.officeforstudents.org.uk/for-students/planning-to-study/study-artificial-intelligence-and-data-science-as-a-postgraduate/

provision with greater number of apprenticeship courses, part-time courses or recognition of micro-credentials that would be attractive to industry professionals or graduates of other subjects looking to re-train. There would need to be sufficient financial support and clear routes into employment to make the transition attractive and viable.

5. Improving the pipeline

Improving the mathematics education pipeline for all is a major undertaking and identifying how this might be achieved is beyond the remit of this report, though much of what is written above will be of relevance to those tasked with orchestrating such improvement. The focus of this study has been to identify and justify potential 'high leverage' interventions whereby motivated individuals and organisations might focus energy and resources on improving an aspect of the mathematics pipeline, and perhaps in particular the mathematics *excellence stream*.

The previous sections of the report have explored a) patterns of excellence and diversity through the pipeline, b) the expert views of published researchers and strategic stakeholders, and c) the mix of recent and ongoing interventions aimed at enhancing varied aspects of the mathematics pipeline. All of this brings us to the vexed 'so what?' question of what to do next.

The purpose of this report is not to propose fully specified interventions but rather to identify those zones of the pipeline where interventions might be most beneficial, either for all learners or for those from disadvantaged backgrounds. Furthermore, the typology presented in the previous section offers a means by which proposed or current interventions can be compared and their affordances and constraints understood. Indeed, any interested party looking to invest further in mathematics education might find it useful to use those dimensions to frame their ambitions clearly.

As discussed in the Introduction, the design of any new intervention or initiative depends not only on the resources available to an organisation but also the values and motivations of its designer/sponsor[69]. In Section 1.1, three general drivers or purposes for mathematics education were identified and it is instructive to consider how existing interventions and initiatives map onto these. Whilst different motivations for interventions are not mutually exclusive, they are found in different balances, and it appears that those organisations with similar values and motivations are more likely to be associated:

- 'Becoming an employee': These interventions and initiatives are motivated in particular by the needs of the workforce and include, for example, work-based mentors and careers guidance: Maths4Girls, Mathematics in Education and Innovation[70] and university outreach programmes could be included here.
- 'Becoming a mathematician': This is where *mathematics for its own sake* is particularly salient. Enrichment, 'thinking like a mathematician', and 'finding your tribe' are particularly important. Examples of initiatives here include UKMT competitions, NRICH problems, 'maths circles'[71], maths schools and the Protect Pure Maths[72] campaign.
- 'Becoming a citizen': There are fewer interventions and initiatives that focus on developing mathematical/statistical literacy for future citizens (as part of the *excellence stream*), though

[69] One framework that has been used to understand the drivers of educational change was that of Raymond Williams. This was later adapted by Paul Ernest (1991) in the context of mathematics education when he delineated the *industrial trainers, technological pragmatics, old humanists, progressive and public educators* and how these various groups' interests would play out in different aspects of mathematics education. It is not difficult to evidence these motivations in the various policies and interventions in mathematics education discussed herein.
[70] MEI is engaged with a wide range of important initiatives in mathematics education (e.g. AMSP); the original 'I' being industry, reflecting the early motivations of the project and its commitment to the mathematics needed by those progressing to other areas of science and employment where advanced mathematics could be applied.
[71] https://mesme.org/maths-circles/
[72] https://www.protectpuremaths.uk/

this would be a core mission of National Numeracy[73], or Young Enterprise[74], for example. This might be due to the poor alignment of decision makers' and funders' goals with the motivations of a mathematics for critical citizenship[75].

In reality, motivations are a blend of the above and, importantly, these intersect with motivations to diversify the mathematics pipeline specifically (e.g. Levelling Up: Maths[76]) and the higher educational pipeline generally (e.g. Into University, Brilliant Club) with implications for mathematics. Understanding these blends of motivations, and whether and how they resonate with other drivers in the education system at large, or of other interventions in mathematics education, is important.

Our work has highlighted two broad areas - each with sub-areas - that would be worth considering for further intervention. In each of these, there is a question of how any new intervention would work within the evolving school and university landscape, for example in relation to maths schools, MATs, etc. Furthermore, consideration needs to be given to whether an intervention might be piloted at a particular scale (e.g. a Priority Educational Investment Area or in a large MAT) or be scattered and engaged with in a more voluntarist way.

The proposed priority areas for intervention do not include a particular focus on early years or primary mathematics education. Ensuring high quality mathematics education in this phase is clearly important given that this establishes patterns of engagement and progress along the mathematics pipeline, but the evidence suggests that there is relatively strong provision at this stage and that the attainment gap is comparatively narrow. More pressing is 1) what happens at Key Stage 3 in terms of engagement and progression and 2) how increased and diversified engagement in post-compulsory mathematics – in A level, undergraduate and postgraduate level – can be secured.

5.1 Possible areas for intervention

5.1.1 Improving engagement and progression in Key Stage 3

The pipeline analysis, extant research and interviews provide considerable support for looking to enhance (or retain) motivation for learning mathematics through Key Stage 3, particularly from students from disadvantaged backgrounds. Interventions should focus on enriching, inspiring, engaging and 'tribe'-forming. They should help to create mathematician identities, thereby keeping more promising mathematicians in the *excellence stream*, increasing the chances of them doing well at GCSE higher tier and be in a position to transition to A level Mathematics. Several of the organisations already committed to this work either have limited *coverage* (e.g. maths schools) or are not strongly tied into the existing school structures (e.g. NRICH) though these and others do sterling work.

Several of the interviewees recognised that the school curriculum was a limiting factor and that some additional activity would be needed. That said, the evolving schooling landscape (i.e. with increasing numbers of large MATs) does offer some opportunities for piloting innovations that can piggy-back on emerging structures and networks. Three more specific ideas are proposed within the area of Key Stage 3 motivation, the first being more teacher-focused and the other two more student focused.

73 https://www.nationalnumeracy.org.uk/
74 https://www.young-enterprise.org.uk/
75 Though such a project is urgently needed in these times, and not just at a basic level but for all, irrespective of role and responsibility in society.
76 https://www.lms.ac.uk/news-entry/19022021-1416/levelling-maths

1. **Specialist teacher programmes** at Key Stage 3. Such programmes would focus on teacher development for improving progression for 11–14-year-olds with mathematical potential, thereby helping to ameliorate the well-documented Key Stage 3 'dip' which is experienced disproportionately by more disadvantaged learners. There are two overlapping problems here: a) upskilling non-specialist teachers and b) curriculum enrichment for lead teachers. The specialist teacher programmes would:
   a) Develop, pilot and evaluate new professional development and accreditation opportunities primarily for non-specialist teachers of mathematics at Key Stage 3, in order to improve subject and pedagogical knowledge. These could be supported through establishing a bursary scheme targeted at either Opportunity/Investment Areas, schools across the country, or in targeted areas which have been identified as facing particular difficulties with recruitment.[77]
   b) Develop, pilot and evaluate professional development programmes focused on the kinds of problem solving, enrichment, 'thinking like a mathematician' and 'nice problems'[78] that many of the interviewees spoke about. The goals of the programme would be to i) inspire experienced teachers of mathematics by re-engaging them with mathematics, ii) develop teaching approaches that improve student engagement and enjoyment of mathematics.

   Either of these could be piloted in several priority areas and in collaboration with a consortium of local partners (e.g. a maths school, university, etc).

2. **Maths clubs and competitions**. There are already various forms of mathematics clubs (e.g. maths circles) and competitions (e.g. UKMT) but this proposal is for the development of a more systematic approach to using such tools to enhance motivation for potential future mathematicians (known or emerging) in Key Stage 3. There may be some merit in developing a programme that makes a virtue of large MATs (and maths leads), encourages student collaboration and provides a framework for individual mathematics enthusiasts to 'find their tribe' and engage in extra curriculum mathematics activities and competitions.

3. **Virtual maths schools** with area-based outreach and engagement programmes. The maths schools are developing outreach programmes that align very well with this area of Key Stage 3 motivation/enrichment. They are, however, limited in their reach and capacity. One way forward would be to develop proposals for virtual maths schools, perhaps in collaboration with existing maths schools, which can offer more comprehensive reach as part of a larger regional collaboration. These could be targeted at this critical Key Stage 3 phase, and for schools serving more disadvantaged communities, but also reach out as appropriate to upper primary students. Recent development in remote provision and online learning present new possibilities in this area.

Most of the above include recommendations to work in geographic areas or across school networks (e.g. PEIAs or MATs) as a basis for pilots. The aim would be to throw a net over an established 'area' with the intention of catching/retaining those who, without support and nurturing at this critical stage of the pipeline are likely to drift away from the mathematics *excellence stream*. There is a risk that such

[77] This would go beyond, in terms of scope and timescale, what is available to participants in the unaccredited Maths Hubs workgroup which currently addresses this need (see https://www.ncetm.org.uk/maths-hubs-projects/specialist-knowledge-for-teaching-mathematics-secondary-non-specialist-teachers-programme/),

[78] 'Nice problems' include those that stimulate mathematical curiosity and enjoyment, that might be drawn from different cultures, both past and present, e.g. like those set by Alex Bellos in The Guardian. https://www.theguardian.com/science/series/alex-bellos-monday-puzzle

interventions simply create more noise in the system or that they only benefit advantaged students. A more systematic approach could aim to increase *concentration* over the *medium term* in more *continuous* ways, targeting socio-economically disadvantaged students.

5.1.2 Improving participation in mathematics post-16

Although this set of proposals focuses on improving post-16 participation (in A level, undergraduate and postgraduate study), some of the suggested interventions must necessarily take place in multiple phases of the mathematics pipeline. It is important to view some of these strategies as continuous across time (i.e. not split into pre-HE and post-HE phases) and space (i.e. avoiding fragmentation in local areas and regions). Indeed, the need for greater coordination is discussed further below.

As above, three broad areas for action are proposed, and the continuity from some of the ideas previous mooted should be clear:

4. **Signalling and career promotion programmes**. One of the challenges here is that mathematics has many areas of application and signalling initiatives targeted at different stages, and for different futures, seem to vary considerably. Promotion of careers for those with undergraduate and postgraduate degrees, and PhD level mathematics, needs improving, and links with industry (e.g. Maths4Girls, Maths Careers) needs wider implementation. The focus in these proposals is information dissemination (i.e. about mathematics/ careers/ disciplines).

    a) Coordinate an information campaign targeted at Year 10 students regarding subject pathways to HE and beyond and the value of mathematics within them. These would need to include a focus on subjects that 15-year-olds would not normally think of as requiring applications of mathematics (e.g. life and social sciences).

    b) Design and develop a YouTube channel that connects big mathematical ideas and applications, whilst also introducing professional mathematicians, users of mathematics and mathematical influencers from different contexts and with diverse backgrounds. This might be similar to the widely acclaimed Periodic Table of Elements[79]. The goal would be for all teachers to know of, and use, the resource to make mathematics, mathematically demanding education pathways and mathematical careers more attractive.

5. **Innovations to tackle social/cultural barriers**. There is ample evidence of how students' attitudes to mathematics impacts choice patterns and addressing such negative attitudes should not start in upper secondary school. An important feature of interventions here is to enable students from underrepresented backgrounds to identify with being mathematical, this should be done through relatable role models that are representative of their own diverse social and cultural heritages.

    a) Transition support and mentoring schemes that support underrepresented groups to navigate the move to university are already established. They might be long term and generic (e.g. Into University), include a mathematics-specific element (e.g. Brilliant Club) or be closely targeted at mathematical transition (e.g. Levelling Up: Maths). Such additional schemes would probably be unhelpful, though further support for the best of these (i.e. based on robust evaluation) is recommended to increase their *concentration*.

[79] https://www.nottingham.ac.uk/periodicnottingham/elements.aspx

b) Enhancing girls' participation at A level is a vexed problem, something which Maths4Girls is taking on by targeting students at Key Stage 3, albeit in a limited way given the scale of the challenge. Female participation is further reduced at undergraduate level and then again at postgraduate level. This requires longitudinal research and development and some suggestions for this are made in the final section. One point made in the report is that the nature of the curriculum and qualifications, both at A level and beyond, might need some reconsideration, perhaps as part of meeting Prime Minister Sunak's maths-to-18 ambitions.

6. **Becoming a mathematician projects**. Not all of those who study A level Mathematics will progress to undergraduate studies in mathematics (or other mathematically demanding courses). Similarly, not all undergraduate mathematicians progress to postgraduate study or become professional mathematicians. Nevertheless, some stakeholders have reported that, at the transition to undergraduate mathematics, there are many students who have not really understood the nature of the discipline. Some organisations are already working on this problem (e.g. AMSP, NRICH, UKMT) but access to such opportunities could be expanded. Mapping and scaling-out this activity would be a good next step and working with a coalition of interested parties to develop a more comprehensive and accessible approach to this problem is needed, perhaps through leveraging widening participation budgets from universities.

5.1.3 Coordinating interventions

The mathematics pipeline in England has very many moving parts. From the perspective of both individuals and institutions, it is not always clear how to navigate this complexity, or whether those learners who could benefit most from various interventions can access them. There is a need for better coordination, perhaps in the form of interventions which have coordination as their primary goal.

This project has included some exploration of one such approach to the coordination of mathematics education support, the Stoke-on-Trent Mathematics Excellence Partnership, and in the current context this innovation warrants some discussion. The MEP aims to coordinate expertise within a hitherto underperforming Opportunity Area to improve the mathematics outcomes of all, and it is enjoying some success. We have investigated this initiative a little and seen how, with expert coordination of existing 'assets' and an inclusive commitment to area-based collaboration, a diverse range of expertise can be brought to bear on the systematic improvement of the mathematics pipeline. However, further investigation is needed to properly understand the extent of the effect and the causal mechanisms.

As with many of the suggestions in this section, it is worth considering how the evolving schooling landscape can be used to ensure that interventions aimed at encouraging potential future mathematicians from disadvantaged backgrounds don't miss the very people for whom they are intended. To that end, several of the proposed areas for intervention might be profitably mapped onto, or implemented through, established and emerging educational structures, i.e. those already coordinating education improvement processes. For example, the strong MATs envisaged in the 2022 White Paper could be an ideal scale to coordinate multiple interventions even if they do have different drivers and objectives.

In a different phase of the mathematics pipeline, there is currently little coordination between universities regarding their Widening Participation outreach programmes. AccessHE[80] in London is an example of such a collaboration. A potential model is for every school to be partnered with specific universities for support with regards to mathematics education. There is evidence of coordinated outreach programmes in some cities (e.g. London, Sheffield, Birmingham) but this approach is not universal (e.g. Nottingham), runs the risk of overlooking rural and coastal areas with lower population densities, and is not specific to mathematics. A good outcome would be for university mathematics departments to coordinate their support for schools and properly evaluate their impact.

[80] https://www.accesshe.ac.uk/

6. Conclusions

6.1 The mathematics pipeline

The mathematics education pipeline in England is long and multifaceted. It comprises millions of students in tens of thousands of schools, colleges and universities who are taught by over a quarter of a million teachers and lecturers. Mathematics education prepares young people for further study, work and citizenship to varying degrees with students' mathematical engagement, progress, attainment and participation at advanced and higher levels patterned in interesting ways. This report has explored those patterns, presented a synthesis of the latest thinking from research and expert stakeholders on the systemic causes of these patterns, and discussed interventions and initiatives intended to improve the mathematics pipeline.

Of particular concern in this report has been what might be termed the *excellence stream*; that part of the mathematics pipeline which includes students with the capabilities to progress to advanced level and university mathematics. This *excellence stream* diminishes over time, during different educational phases and at different rates, for students from diverse backgrounds, and with varied causal mechanisms. The data analysis focused on the flow of student cohorts through the mathematics education pipeline (section 2), identifying features of, and influences on, the pipeline (section 3). In light of these, the team considered the interventions that students might encounter and the extent to which these might help to improve engagement, progression, attainment and thereby participation (sections 4 and 5).

The project team discussed at length the problems of the metaphoric language of pipeline[4], in particular the dehumanising of students. Whilst there is considerable merit in thinking of mathematics education in system terms, and of the adjustments that could be made to enable the system to function more effectively – particularly for those seeking to act at scale - the language of pipeline emphasises force and flow and, arguably, diminishes student agency (c.f. 'pathways', for example, which imbue a sense of choice). One must not forget that for many students, particularly the most disadvantaged students in the *excellence stream*, their most pressing challenges might be the cost-of-living impacting their families, having a quiet place to work, enough to eat, and so on.

As part of the project, the team explored the value of developing cases of students' mathematics education journeys. Such an approach can imagine a *desirable* mathematics education experienced for a student with particular characteristics, one that leads to advanced mathematics study perhaps, and contrast that with the more *typical* experiences of students from similar backgrounds. This is a perennial problem for research and policymaking, bridging between the system and the individual; between the averages that describe the whole but do not fit anyone, to the unique stories of individuals, the sum of which does not add up to the whole.

The focus herein, on the *excellence stream* of the mathematics pipeline, offers a great deal of interesting detail in its flow, as seen in the Sankey diagrams in Section 2. Many aspects of the pipeline have been described above but four features are particularly noteworthy and pertinent to the characteristics of the *excellence stream* here:

- **The Key Stage 3 attainment gap** opens up following the transition from primary to secondary school thereby losing many students from more economically disadvantaged backgrounds from the *excellence stream* through to GCSE. For those who remain, and attain highly at GCSE, they are equally likely to progress to advanced mathematics;

- **Asian students' mathematics progress** through secondary schooling, and their subsequent attainment and participation in advanced mathematics is striking in its positive divergence from the trends of other ethnic groups.
- **Girls' participation in A level Mathematics** continues to be lower than that of boys (and even more so in Further Mathematics) and this trend continues into undergraduate study. These longstanding patterns are stubbornly resistant to efforts to change them. Ameliorating girls' loss from the mathematics *excellence stream* needs new approaches that happen early in secondary education, and possibly before that.
- In the **transition to university mathematics** economically disadvantaged students are more likely than their middle-class peers, for similar prior attainment, to progress to undergraduate mathematical study. The corollary of this is that many are lost from the mathematics *excellence stream* to the life and social sciences. However, once at university, economically disadvantaged students are less likely to complete their degree.

Two of the above are about choices. Students do not generally choose *mathematics* versus *not-mathematics* but rather *mathematics* versus *another subject*. Influencing those choice patterns needs to happen in multiple ways and at every stage along the pipeline. Better, holistic understanding of the genesis of those choice patterns is needed in order to inform better interventions that improve participation. This is a long-term project that requires 'cathedral thinking' and sustained investment, and that is made all the more challenging by changing political priorities. However, as with all complex systems, it is not clear what combinations of actions will yield such changes in choices or other improvements in the pipeline, nor indeed which will unintentionally disrupt the flow.

Designing effective interventions, initiatives and policies is notoriously challenging though there is growing expertise on how to do this. Similarly, the need for robust evaluation of any interventions so as to build the knowledge base is important. All of this is, of course, easy to state as many have done - and difficult to do.

6.2 Setting a research agenda

This report reflects the complex amalgam of pipeline patterns, their causes and effects. It has also considered some of the well-intentioned interventions and initiatives designed to improve engagement, progression, attainment and participation in mathematics. There is presently a weak evidence base on the effectiveness of these interventions and initiatives, either individually, or in combination. There is an ongoing need for more robust approaches to evaluation, and for the improved, research-based design of interventions.

Some possible interventions that were not included above fit more appropriately in a section on research. In a parallel to the development of medicines, the first step is to invest in the design of new treatments, taking time to understand the causal mechanisms and developing clear proof of concept. Only thereafter can one trial and explore any other 'side effects' of such 'treatments'. Such an approach is costly and requires teams of decision makers, researchers, designers, trainers and practitioners to work together in well-orchestrated ways (see Burkhardt & Schoenfeld, 2020).

Some possible areas of research include:

- How different pedagogic and assessment approaches encourage greater engagement and later participation.
- Whether well-designed and sustainable area-based coordination programmes can increase the *excellence stream*.
- Understanding of the process by which some groups of students (or schools) buck the trends and increase progress, attainment, and participation.
- Understanding the emergence of preferences for postgraduate study through undergraduate programmes; variations by demographics, institutions, etc. and the impact of interventions.
- Mapping of access to interventions and initiatives, individually or combined, by background, school type, area, additional support (e.g. PEIA) etc.
- How the process of designing mathematics interventions can be improved by implementation theory to enhance evaluation and scaling.
- Longitudinal cohort studies of students to understand the trajectories of individuals and groups of students over time.

At present, research expertise is too diffuse with limited capability within the system to orchestrate the necessary experts. Furthermore, communication and collaboration between policymakers, researchers, practitioners and other stakeholders is fragmented. Whilst there is further evaluation, research, and development that could be usefully undertaken on individual interventions and initiatives, a more ambitious plan would be to establish a well-funded *centre for longitudinal studies of mathematics education* that can take a holistic, systemic view of the pipeline and the policies, interventions and initiatives within it.

The Government's industrial strategy aims to achieve an R&D spend of 2.4% of GDP by 2028. Whilst there is a real commitment to such investment in health, for example, such investment in mathematics education pipeline R&D (reception to postgraduate) falls way short of this. Without such investment, long-term improvement at the system level is unlikely to be realised. Nevertheless, interventions that demonstrate the potential impact of a coordinated implementation, evaluation and scaling of research-based, expertly-designed educational products and processes are much needed. It is our hope that this report can contribute in some way to that endeavour.

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8. Appendices

8.1 The team

Andy Noyes is Professor of Education at the University of Nottingham where he has been Head of the School of Education and APVC for Research in the Faculty of Social Sciences. Andy is Chair of the Joint Mathematical Council of the UK and a member of the Royal Society's Advisory Committee on Mathematics Education. His research interests centre on post-16 education, on change in complex systems and on educational policy. He has advised DfE on post-16 mathematics and worked with regulators in England and Scotland on qualifications reform.

Chris Brignell is Associate Professor of Statistics at the University of Nottingham where he is Head of Mathematics Education and Scholarship in the School of Mathematical Sciences. His research interests focus on pedagogical approaches and student engagement in higher education mathematics, as well as interdisciplinary science education and education for sustainable development. He is an accredited ONS researcher and supported the secondary analysis of the National Pupil Database and HESA data on this project.

Laurie Jacques is one of the project's research associates. She studied undergraduate mathematics and has an MA in Mathematics Education. Formerly a primary teacher (1998-2009), she has been a member of the Advisory Committee on Mathematics Education (2005-2010), panel member of the Williams Review (2008) and Director for Policy and Quality (and later for Primary) for the NCETM (2009-2013). She now works as a teacher educator and doctoral researcher in mathematics education with research interests in policy implementation from a classroom perspective and teaching mathematics for equitable classrooms.

Jake Powell is one of the project's research associates. He is a recent example of someone who has gone through all stages of the mathematics pipeline in England having recently been awarded his doctorate in mathematics at the University of Nottingham. Previously, Jake completed an integrated master's in mathematics degree at the University of Warwick in 2016. He is an accredited ONS researcher and led the secondary analysis of National Pupil Database and HESA data in this project.

Mike Adkins is a Senior Research Fellow in Education at the University of Nottingham. His research interests focus on school effects, inequalities in educational participation and higher education transitions, particularly from a maths and science education perspective. His expertise lies in the application of advanced statistical methods to very large scale administrative social science datasets and the running of randomised controlled trials across primary, secondary and further education. He is an accredited ONS researcher and supported the secondary analysis of the National Pupil Database and HESA data on this project.

8.2 Typology of mathematics interventions (full version)

This fuller version of the typology of mathematics interventions in Section 4 adds definition and some examples of how this might work for a few interventions ('X' denotes priority category and 'x' additional categories, as appropriate). The team have undertaken an (unverified) analysis across a wide range of interventions and initiatives in mathematics education and such an approach might be useful to identify areas for new activity or for the scale up of existing programmes.

Dimension	Category	Definition	Int 1	Int 2	Int 3	Etc.
Aim (1 or more) What are the intervention's main aims?	Engagement	To enhance student enthusiasm for, interest in, and attitudes to mathematics	X			
	Progression	To enable students to make good progress and to support transitions	x	x		
	Attainment	To enhance student performance in the high stakes assessment		X		
	Participation	To encourage participation in mathematics post-16 or at university		x	X	
Phase (1 or more) How old are the participants?	Recep/Primary	4-11: students in reception, Key Stage 1 and/or Key Stage 2.	X			
	Secondary	11-16: students in Key Stage 3 and/or 4.	X	x	X	
	Post-16	16-19: students in Key Stage 5		X	x	
	Undergraduate	18-21+: students on undergraduate BSc, MMath courses.		x		
	Postgraduate	21+: students on MSc or PhD courses.				
Attainment (select 1) What is the typical mathematical ability of targeted participants?	High-attainers	High mathematical attainment (top quartile)		X	X	
	Low-attainers	Low mathematical attainment (bottom quartile)				
	Universal	Any level of mathematical attainment.	X			
Demographics (1 or more) What are the demographics of the participants?	Income	Students from low socio-economic backgrounds			x	
	Gender	Students of a particular gender or sexuality.			X	
	Ethnicity	Students of a particular ethnicity.				
	Universal	All students, regardless of characteristics	X	X		
Coverage (select 1) Where are the participants?	National	Multiple locations, either connected directly to the intervention's centre or through a coordinated network of sites	X	X	X	

	Local	In a local area such as a LEA, MAT, or local hub with connections to other organisations				
	Site	Typically in an individual institution, such as a school or university				
Concentration (select 1) What proportion of target audience participate?	High (50%+)	The majority of *eligible*[81] participants engage.		X		
	Mid (10-50%)	A sizeable minority of *eligible* participants engage.				
	Low (<10%)	A small minority of *eligible* participants engage.	X		X	
Dosage (1 in each) i) Over what period is the intervention? ii) How frequent is the intervention?	Short-term	One term or less (e.g. a residential summer school).			X	
	Medium-term	Within a particular phase or during a period of 1/2 years		X		
	Long-term	An extended period covering more than one phase of education	X			
	Continuous	Participants regularly and frequently engage during the intended timescale		X		
	Intermittent	Participants dip in-and-out of the intervention, or engage with the intervention infrequently over an extended period.	X		X	
Model for delivery (select 1) How is the intervention facilitated?	Teacher PD	By developing teachers' knowledge and skills, and aid retention		x		
	Teacher-directed	Through school or university staff (e.g. in a classroom)	X	X		
	Self-directed	Participants access the intervention on their own (e.g. on a digital platform)				
	Other-directed	Intervention is facilitated by an external provider (e.g. an education charity)		x	X	

[81] 'Eligible' here is loosely defined as those in the target *phase*, *attainment* level, *demographic* characteristics and area (i.e. *coverage*). For the Stoke MEP, eligibility would therefore be all of the students in the OA, and perhaps more besides (i.e. in linked MAT schools just outside of the area) and *concentration* for this intervention would be 'high', at or near 100%. In contrast, eligibility for a maths school is limited, to varying extents, by area, demographics, attainment, etc. Some of the maths schools have many strong applicants per place, and there are probably many more eligible students who do not apply, so the *concentration* in this instance would be 'low'.

8.3 Tables

8.3.1 Notes on Tables

1. Tables compiled using data supplied by Department for Education (National Pupil Database) and Higher Education Statistics Agency.
2. All counts below 10 are rounded down to zero. All other counts are rounded to the nearest 5.
3. Cohort 1 consists of 592,885 students who took GCSEs in 2016/17. We have complete academic records, including attainment at Key Stage 1, Key Stage 2 and A level for 474,515 of these students.
4. Cohort 2 consists of 258,400 students who started an undergraduate degree programme in 2015/16 and had previously taken GCSEs and A levels.
5. Notation such as 3+ or 6- means levels or grades 3 and above or 6 and below respectively.
6. Key Stage 2 level 5 is sub-divided into upper level 5 (5U) and lower level 5 (5L) based on student's raw scores to give greater granularity.
7. Where a student has attempted a qualification on more than one occasion then we use the highest grade they achieved.
8. The NPD uses the label gender, rather than sex, to indicate whether a student is male or female.
9. Socio-economic status is measured using the Income Deprivation Affecting Children Index (IDACI) at the end of Key Stage 4, a student's free school meal (FSM) eligibility, and socio-economic class (SEC) as recorded by HESA.
10. IDACI scores are based on students' postcodes and reported by quintile. Quintile 1 is the quintile with the greatest income deprivation.
11. SEC is based on the occupation of the highest earning parent or guardian. Classes 1 and 2 are higher and lower, respectively, managerial and professional occupations; classes 3 and 4 are intermediate occupations and classes 5 and above are technical, semi-routine and routine occupations or long-term unemployed.
12. Undergraduate subject choices are based on the JACS3.0 (2015/16-2018/19) and CAH-01 (2019/20) classification system.
13. A student is deemed to be studying mathematics if their undergraduate course classification is 100% CAH09 (mathematical sciences) or joint mathematics if mathematical sciences comprises more than 30% of their course classification.
14. For undergraduate courses not classified as mathematics or joint mathematics, we sub-divide them into those which are associated with mathematics (e.g. physics, engineering, computing) and those which contain low amounts of mathematics (e.g. languages, history, creative arts).

8.3.2 Cohort 1 transitions

Key Stage 1 level	A level Maths	Key stage 2 level 3-	4	5L	5U	6	Total
3+	Yes	0	400	4605	16355	9530	**30890**
	No	80	11325	26760	28255	4565	**70985**
2A	Yes	0	1520	5720	6455	1085	**14780**
	No	1695	59290	42025	16160	745	**119915**
2B	Yes	10	1600	2355	1275	120	**5360**
	No	10565	83215	18360	3725	85	**115950**
2C-	Yes	45	820	510	155	10	**1540**
	No	49875	60370	4290	555	10	**115100**
Total		**62270**	**218540**	**104625**	**72935**	**16150**	**474520**

Table 1A. Number of students in Cohort 1 by Key Stage 1 and Key Stage 2 Mathematics level, and whether they would later take A level Mathematics

Key Stage 2 level	A level Maths	GCSE grade 6-	7	8	9	Total
6	Yes	80	905	3460	6300	**10745**
	No	1415	1640	1665	680	**5400**
5U	Yes	1060	5875	10550	6755	**24240**
	No	28310	13235	6065	1085	**48695**
5L	Yes	1715	5340	4890	1245	**13190**
	No	76165	12120	2965	190	**91440**
4	Yes	1235	1995	990	125	**4345**
	No	208990	4545	650	10	**214195**
3-	Yes	35	15	0	0	**50**
	No	62175	35	0	0	**62210**
Total		**381180**	**45705**	**31235**	**16390**	**474510**

Table 1B. Number of students in Cohort 1 by Key Stage 2 Mathematics level and GCSE Mathematics grade, and whether they would later take A level Mathematics

GCSE grade	A level Mathematics status: No Mathematics	Mathematics only	Maths & Further maths	Total
9	1970	8800	5625	**16395**
8	11340	17815	2080	**31235**
7	31580	13830	300	**45710**
6-	377050	4100	20	**381170**
Total	**421940**	**44545**	**8025**	**474510**

Table 1C. Number of students in Cohort 1 by GCSE Mathematics grade, and whether they would later take A level Mathematics or both A level Mathematics and A level Further Mathematics

8.3.3 Cohort 2 transitions

GCSE grade	Maths Degree	A level outcome: No Maths	Maths C-	Maths B	Maths A/A*	A*A* or A*A	Total
A*	Yes	70	165	445	2050	2170	**4900**
	No	11650	4050	6640	16670	3695	**42705**
A	Yes	60	330	480	710	110	**1690**
	No	40295	10850	5985	3990	145	**61265**
B	Yes	65	95	70	45	0	**275**
	No	64040	2995	645	280	0	**67960**
C	Yes	0	0	0	0	0	**0**
	No	63995	100	20	10	0	**64125**
D-	Yes	0	0	0	0	0	**0**
	No	14805	0	0	0	0	**14805**
Total		**194980**	**18585**	**14285**	**23755**	**6120**	**257725**

*Table 2A. Number of students in Cohort 2 by GCSE Mathematics grade and A level Mathematics result, and whether they would later complete a degree in Mathematics. Note "A*A* or A*A" means a student achieved a minimum of A*A across A level Mathematics and A level Further Mathematics.*

A level outcome	Maths Degree	Undergraduate course type - started: Low maths	Associated	Joint maths	Mathematics	Total
A*A* or A*A	Yes	10	60	505	1705	**2280**
	No	770	2870	65	135	**3840**
Maths A/A*	Yes	50	80	855	1815	**2800**
	No	10725	9740	210	275	**20950**
Maths B	Yes	30	30	245	695	**1000**
	No	7440	5620	75	150	**13285**
Maths C-	Yes	20	35	145	390	**590**
	No	10980	6760	120	135	**17995**
No Maths	Yes	30	0	105	60	**195**
	No	176295	18140	320	30	**194785**
Total		**206350**	**43335**	**2645**	**5390**	**257720**

*Table 2B. Number of students in Cohort 2 by A level Mathematics result and the type of undergraduate course started, and whether they would later complete a degree in Mathematics. Note "A*A* or A*A" means a student achieved a minimum of A*A across A level Mathematics and A level Further Mathematics.*

Course started	Undergraduate course type - completed: No degree	Low maths	Associated	Joint maths	Mathematics	Total
Mathematics	535	110	80	100	4560	**5385**
Joint maths	315	290	190	1710	145	**2650**
Associated	6850	2015	34265	85	125	**43340**
Low maths	26800	178345	1065	70	70	**206350**
Total	**34500**	**180760**	**35600**	**1965**	**4900**	**257725**

Table 2C. Number of students in Cohort 2 by the type of undergraduate course started and completed

8.3.4 Cohort 1 attainment by demographic

		Key Stage 1 level					No level	
		2C-	2B	2A	3+	Total	recorded	Total
Gender	Male	71550	61770	68035	62640	**263995**	39775	**303770**
	Female	63150	66735	74510	47890	**252285**	36830	**289115**
Ethnicity	White	96650	96865	110275	85990	**389780**	24965	**414745**
	Mixed	5900	5390	5930	4340	**21560**	2710	**24270**
	Black	8290	6270	5415	2555	**22530**	6120	**28650**
	Asian	14855	11975	11540	7690	**46060**	9305	**55365**
	Unknown	9005	8000	9385	9950	**36340**	33505	**69845**
IDACI quintile	5th	14800	20495	28075	28020	**91390**	6735	**98125**
	4th	22200	26075	31855	27365	**107495**	7735	**115230**
	3rd	23895	23660	26240	19415	**93210**	8725	**101935**
	2nd	31940	26620	26240	15955	**100755**	12250	**113005**
	1st	36680	26295	23105	11530	**97610**	11965	**109575**
	Unknown	5185	5355	7030	8245	**25815**	29195	**55010**
FSM	No	80490	93215	113240	96685	**383630**	65125	**448755**
	Yes	54205	35285	29310	13840	**132640**	11485	**144125**

Table 3A. Number of students in Cohort 1 by Key Stage 1 Mathematics level and demographic

		Key Stage 2 level						No level	
		3-	4	5L	5U	6	Total	recorded	Total
Gender	Male	41445	113320	58585	44285	12170	**269805**	33970	**303775**
	Female	39695	123785	53705	35190	6435	**258810**	30305	**289115**
Ethnicity	White	59355	179725	85215	60250	12500	**397045**	17700	**414745**
	Mixed	3580	10115	4745	3230	850	**22520**	1755	**24275**
	Black	4850	12390	4705	2370	460	**24775**	3880	**28655**
	Asian	7805	21905	10540	7170	2365	**49785**	5585	**55370**
	Unknown	5550	12970	7085	6455	2430	**34490**	35360	**69850**
IDACI quintile	5th	8885	36225	23270	19390	5385	**93155**	4975	**98130**
	4th	13565	47570	25255	18930	4535	**109855**	5385	**115240**
	3rd	15070	43810	20315	13990	2930	**96115**	5820	**101935**
	2nd	19330	50460	20315	12445	2270	**104820**	8185	**113005**
	1st	21210	51020	18200	9795	1465	**101690**	7875	**109565**
	Unknown	3085	8015	4935	4925	2020	**22980**	32030	**55010**
FSM	No	47105	167745	90520	68615	17115	**391100**	57660	**448760**
	Yes	34035	69355	21770	10860	1490	**137510**	6615	**144125**

Table 3B. Number of students in Cohort 1 by Key Stage 2 Mathematics level and demographic

		GCSE grade						No grade	
		5-	6	7	8	9	Total	recorded	Total
Gender	Male	216585	29410	26105	18815	11280	**302195**	1570	**303765**
	Female	206420	30055	26065	17390	8095	**288025**	1085	**289110**
Ethnicity	White	294345	43885	36940	24725	12925	**412820**	1925	**414745**
	Mixed	16915	2530	2180	1615	885	**24125**	145	**24270**
	Black	21185	2855	2505	1520	485	**28550**	105	**28655**
	Asian	33405	6390	6685	5405	3375	**55260**	110	**55370**
	Unknown	57155	3805	3865	2945	1705	**69475**	370	**69845**
IDACI quintile	5th	55050	13670	13055	9940	6230	**97945**	175	**98120**
	4th	74625	13975	12525	8770	5005	**114900**	340	**115240**
	3rd	72335	10770	9030	6280	3150	**101565**	365	**101930**
	2nd	85750	10310	8470	5470	2380	**112380**	625	**113005**
	1st	88500	8330	6650	3795	1455	**108730**	845	**109575**
	Unknown	46745	2415	2440	1945	1155	**54700**	305	**55005**
FSM	No	302015	49990	45215	32330	17950	**447500**	1255	**448755**
	Yes	120990	9480	6955	3875	1430	**142730**	1395	**144125**

Table 3C. Number of students in Cohort 1 by GCSE Mathematics grade and demographic

		A level Mathematics grade						No Maths	
		D-	C	B	A	A*	Total	A level	Total
Gender	Male	10405	7075	7660	10885	8185	**44210**	259555	**303765**
	Female	6450	4890	5245	7175	3920	**27680**	261435	**289115**
Ethnicity	White	9190	6585	7015	9395	6255	**38440**	376310	**414750**
	Mixed	635	500	485	695	480	**2795**	21475	**24270**
	Black	1115	640	585	600	250	**3190**	25465	**28655**
	Asian	3290	2110	2130	2820	1615	**11965**	43400	**55365**
	Unknown	2625	2130	2695	4550	3505	**15505**	54335	**69840**
IDACI quintile	5th	3170	2650	2915	4535	3200	**16470**	81655	**98125**
	4th	3315	2585	2665	3660	2385	**14610**	100625	**115235**
	3rd	2890	1830	1935	2425	1545	**10625**	91310	**101935**
	2nd	3060	1795	1750	2055	1060	**9720**	103290	**113010**
	1st	2410	1340	1335	1375	675	**7135**	102430	**109565**
	Unknown	2010	1760	2305	4015	3240	**13330**	41680	**55010**
FSM	No	14315	10635	11620	16630	11490	**64690**	384065	**448755**
	Yes	2545	1330	1285	1430	615	**7205**	136925	**144130**

Table 3D. Number of students in Cohort 1 by A level Mathematics grade and demographic

		A level Further Mathematics grade						No FM	
		D-	C	B	A	A*	Total	A level	Total
Gender	Male	1020	1100	1640	2530	2320	**8610**	295160	**303770**
	Female	380	435	725	985	730	**3255**	285865	**289120**
Ethnicity	White	850	880	1295	1820	1630	**6475**	408270	**414745**
	Mixed	80	55	105	145	120	**505**	23765	**24270**
	Black	50	55	65	85	35	**290**	28365	**28655**
	Asian	210	260	355	500	370	**1695**	53675	**55370**
	Unknown	210	285	540	965	890	**2890**	66955	**69845**
IDACI quintile	5th	245	325	550	850	865	**2835**	95290	**98125**
	4th	320	350	510	700	605	**2485**	112750	**115235**
	3rd	245	220	370	480	375	**1690**	100240	**101930**
	2nd	245	240	290	375	220	**1370**	111640	**113010**
	1st	175	160	175	220	150	**880**	108690	**109570**
	Unknown	165	240	465	895	835	**2600**	52415	**55015**
FSM	No	1210	1345	2180	3310	2915	**10960**	437800	**448760**
	Yes	190	190	185	205	135	**905**	143225	**144130**

Table 3E. Number of students in Cohort 1 by A level Further Mathematics grade and demographic

8.3.5 Cohort 2 attainment by demographic

		GCSE grade							
		F-	E	D	C	B	A	A*	Total
Gender	Male	905	1185	3560	25725	29460	29785	25285	**115905**
	Female	1450	2080	6120	38425	38775	33175	22320	**142345**
Ethnicity	White	920	1630	5495	44680	50305	47760	35965	**186755**
	Mixed	260	305	810	4480	4125	3515	2660	**16155**
	Black	695	650	1500	6025	4555	2850	1145	**17420**
	Asian	450	665	1820	8805	9035	8505	7150	**36430**
	Unknown	25	20	50	160	215	325	685	**1480**
IDACI quintile	5th	105	285	1010	8925	12000	12450	10050	**44825**
	4th	155	315	1180	10150	12375	11920	8330	**44425**
	3rd	250	550	1620	12055	12780	11090	6875	**45220**
	2nd	485	805	2290	13735	12450	9870	5185	**44820**
	1st	970	1180	2935	15810	12250	8080	3600	**44825**
	Unknown	390	130	645	3470	6385	9545	13565	**34130**
FSM	No	1160	1760	6170	47625	55955	55255	44770	**212695**
	Yes	710	955	2365	13590	9980	6180	2530	**36310**
	Unknown	485	555	1145	2930	2300	1520	305	**9240**
Socio-economic class	1	200	290	1130	9405	14335	17315	18255	**60930**
	2	335	540	1880	13955	16255	15715	11435	**60115**
	3	205	335	1080	7610	7835	6825	4245	**28135**
	4	170	320	845	5335	5195	4250	2450	**18565**
	5	90	150	435	3480	3415	2665	1335	**11570**
	6	415	495	1435	8265	6860	4715	2385	**24570**
	7/8	305	395	955	5805	4550	3025	1285	**16320**
	Unknown	630	740	1925	10305	9785	8445	6215	**38045**

Table 4A. Number of students in Cohort 2 by GCSE Mathematics grade and demographic

		A level Mathematics grade						No Maths	
		D-	C	B	A	A*	Total	A level	Total
Gender	Male	5430	5685	8300	10125	8640	**38180**	77805	**115985**
	Female	3650	3840	6000	7100	4080	**24670**	117750	**142420**
Ethnicity	White	5490	6255	9785	12570	9315	**43415**	143430	**186845**
	Mixed	625	605	880	1010	755	**3875**	12300	**16175**
	Black	735	600	725	545	245	**2850**	14590	**17440**
	Asian	2170	1990	2760	2885	2190	**11995**	24460	**36455**
	Unknown	60	75	150	215	215	**715**	780	**1495**
IDACI quintile	5th	1510	1790	2860	3775	2895	**12830**	32005	**44835**
	4th	1480	1730	2605	3135	2300	**11250**	33185	**44435**
	3rd	1610	1625	2310	2540	1885	**9970**	35265	**45235**
	2nd	1765	1630	2245	2105	1360	**9105**	35725	**44830**
	1st	1865	1580	1915	1625	940	**7925**	36920	**44845**
	Unknown	860	1175	2365	4040	3340	**11780**	22455	**34235**
FSM	No	7345	8175	12715	15920	12020	**56175**	156645	**212820**
	Yes	1370	1145	1400	1165	660	**5740**	30585	**36325**
	Unknown	370	205	180	140	35	**930**	8320	**9250**
Socio-economic class	1	1995	2505	4295	6315	5275	**20385**	40580	**60965**
	2	1985	2195	3395	4025	2945	**14545**	45605	**60150**
	3	1020	1020	1435	1665	1080	**6220**	21925	**28145**
	4	745	725	990	965	615	**4040**	14525	**18565**
	5	405	445	580	530	325	**2285**	9290	**11575**
	6	940	795	1015	960	570	**4280**	20295	**24575**
	7/8	625	545	625	520	315	**2630**	13710	**16340**
	Unknown	1370	1285	1965	2250	1595	**8465**	29615	**38080**

Table 4B. Number of students in Cohort 2 by A level Mathematics grade and demographic

		A level Further Mathematics grade						No FM	
		D-	C	B	A	A*	Total	A level	Total
Gender	Male	750	890	1760	2400	2550	**8350**	107635	**115985**
	Female	230	345	590	960	825	**2950**	139480	**142430**
Ethnicity	White	665	905	1725	2525	2535	**8355**	178490	**186845**
	Mixed	60	70	135	175	200	**640**	15535	**16175**
	Black	30	35	50	65	45	**225**	17210	**17435**
	Asian	195	200	385	530	515	**1825**	34625	**36450**
	Unknown	25	30	50	65	75	**245**	1250	**1495**
IDACI quintile	5th	820	1090	2135	3165	3235	**10445**	202385	**212830**
	4th	130	125	200	185	135	**775**	35555	**36330**
	3rd	30	20	15	10	0	**75**	9175	**9250**
	2nd	185	250	490	780	775	**2480**	42345	**44825**
	1st	190	230	475	650	610	**2155**	42280	**44435**
	Unknown	160	200	395	525	460	**1740**	43490	**45230**
FSM	No	175	205	290	405	325	**1400**	43425	**44825**
	Yes	165	180	260	270	180	**1055**	43785	**44840**
	Unknown	100	165	435	730	1020	**2450**	31785	**34235**
Socio-economic class	1	230	355	745	1330	1500	**4160**	56810	**60970**
	2	205	260	525	800	780	**2570**	57580	**60150**
	3	105	130	240	295	280	**1050**	27090	**28140**
	4	70	65	140	185	140	**600**	17975	**18575**
	5	50	55	95	95	75	**370**	11205	**11575**
	6	95	95	165	175	130	**660**	23915	**24575**
	7/8	55	70	85	80	65	**355**	15980	**16335**
	Unknown	165	205	355	400	410	**1535**	36550	**38085**

Table 4C. Number of students in Cohort 2 by A level Further Mathematics (FM) grade and demographic

		Undergraduate course type - started				
		Low maths	Associated	Joint maths	Mathematics	Total
Gender	Male	76265	34465	1615	3635	**115980**
	Female	130580	9010	1045	1780	**142415**
Ethnicity	White	152765	28285	1735	4060	**186845**
	Mixed	12875	2865	155	275	**16170**
	Black	13835	3310	145	145	**17435**
	Asian	26310	8640	600	900	**36450**
	Unknown	1060	375	25	35	**1495**
IDACI quintile	5th	35595	7560	495	1185	**44835**
	4th	35820	7080	465	1070	**44435**
	3rd	36875	6970	425	965	**45235**
	2nd	36050	7460	450	870	**44830**
	1st	35615	8110	395	720	**44840**
	Unknown	26895	6300	435	600	**34230**
FSM	No	169915	35800	2295	4820	**212830**
	Yes	29455	6060	305	510	**36330**
	Unknown	7480	1625	60	85	**9250**
Socio-economic class	1	47165	11440	790	1575	**60970**
	2	48930	9335	585	1290	**60140**
	3	22860	4440	300	550	**28150**
	4	14835	3195	185	355	**18570**
	5	9310	1935	95	235	**11575**
	6	20060	3875	215	425	**24575**
	7/8	13375	2570	120	265	**16330**
	Unknown	30305	6690	370	720	**38085**

Table 4D. Number of students in Cohort 2 by the type of undergraduate course they started and demographic

		Undergraduate course type - completed					
		No degree	**Low maths**	**Associated**	**Joint maths**	**Maths**	**Total**
Gender	Male	18595	64980	27905	1225	3280	**115985**
	Female	16050	116160	7805	760	1645	**142420**
Ethnicity	White	22400	135620	23725	1410	3690	**186845**
	Mixed	2710	10850	2270	95	250	**16175**
	Black	3845	11045	2335	70	135	**17430**
	Asian	5485	22705	7055	385	820	**36450**
	Unknown	205	920	320	20	30	**1495**
IDACI quintile	5th	4360	32300	6650	405	1115	**44830**
	4th	4815	32240	6025	375	985	**44440**
	3rd	5900	32460	5675	320	880	**45235**
	2nd	7270	30645	5820	305	790	**44830**
	1st	9025	29015	5955	225	625	**44845**
	Unknown	3275	24485	5585	350	535	**34230**
FSM	No	24430	151835	30355	1795	4415	**212830**
	Yes	7610	23750	4345	165	455	**36325**
	Unknown	2605	5560	1010	20	55	**9250**
Socio-economic class	1	5650	43095	10105	630	1485	**60965**
	2	6895	43765	7850	470	1170	**60150**
	3	3755	20065	3595	220	510	**28145**
	4	2660	12970	2505	125	315	**18575**
	5	1545	8170	1570	80	215	**11580**
	6	4100	16995	2970	145	365	**24575**
	7/8	2840	11270	1890	90	240	**16330**
	Unknown	7195	24810	5220	225	630	**38080**

Table 4E. Number of students in Cohort 2 by the type of undergraduate course they completed and demographic

8.3.6 Degree subjects studied by high achieving A level Mathematics students in Cohort 2

		Gender		Socio-economic class					
Subject	**HESA Code**	**Female**	**Male**	**1**	**2**	**3/4**	**5+**	**Unknown**	**Total**
All subjects	**Total**	**11180**	**18765**	**11590**	**6965**	**4325**	**3220**	**3845**	**29945**
Mathematics	CAH09-01-01	1480	3100	1495	1135	715	675	560	**4580**
Medicine	CAH01-01-02	1385	1245	1250	510	320	215	335	**2630**
Economics	CAH15-02-01	625	1695	965	495	325	190	345	**2320**
Physics	CAH07-01-01	520	1725	840	535	355	265	255	**2245**
Mechanical eng.	CAH10-01-02	205	1345	550	345	270	170	210	**1545**
Chemistry	CAH07-02-01	575	780	530	345	170	145	165	**1355**
Computer sci.	CAH11-01-01	135	1070	395	305	185	155	165	**1205**
Chemical eng.	CAH10-01-09	245	780	365	235	160	145	125	**1025**
Biology	CAH03-01-02	390	225	260	140	90	60	70	**615**
Aeronautical eng.	CAH10-01-04	75	535	230	150	90	80	60	**610**
Engineer. (other)	CAH10-01-01	150	450	270	125	85	45	70	**600**
Civil engineering	CAH10-01-07	140	450	210	140	105	75	65	**590**
Electrical eng.	CAH10-01-08	50	470	180	115	75	70	80	**520**
Molecular biol.	CAH03-01-08	290	215	200	115	60	55	70	**500**
Accounting	CAH17-01-08	175	295	135	110	55	50	115	**475**
History	CAH20-01-01	220	235	235	120	40	20	45	**455**
Law	CAH16-01-01	255	200	190	95	60	45	65	**455**
Health sciences	CAH02-06-01	240	190	170	95	65	45	55	**430**
Psychology	CAH04-01-01	300	90	155	95	45	40	50	**390**
Management	CAH17-01-04	145	215	135	85	50	20	65	**360**
Finance	CAH17-01-07	95	255	105	80	50	55	60	**350**
Veterinary med.	CAH05-01-01	240	65	145	75	35	25	25	**305**
Pharmacy	CAH02-02-03	190	110	80	55	60	60	45	**300**
Philosophy	CAH20-02-01	110	190	135	70	35	15	45	**300**
Architecture	CAH13-01-01	155	145	120	60	50	25	40	**300**
Politics	CAH15-03-01	120	175	135	70	30	15	50	**300**

Table 5. Degree subject studied by students in Cohort 2 who achieved grade A or A in A level Mathematics by gender and socio-economic class. Only subjects with 300 or more students shown.*

The project team is grateful for the support of XTX Markets for funding this research. The views expressed within are wholly those of the authors and not necessarily those of the funder.