

Introductory report Concepts and plans



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To find out more about the Observatory's work, visit: www.nottingham.ac.uk/observatory

Cover image:

Percentage of pupils achieving grade 5 or above in Ebacc Mathematics component in 2022/23 by Local Authority. Dark is lower %, lighter is higher %. Source: <u>gov.uk</u>

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Observatory for Mathematical Education

Evidence Excellence Equity

	What's the	e mission?	To generate research to i and longer-	and communicate, evide improve mathematics of term benefits for individ	ence-driven, and policy-relevant education, learner outcomes duals and society.
	Why Nottingham? Announced in November 2023, the Observatory builds upon the renowned work of the Shell Centre for Mathematical Education , founded in 1967 at the University of Nottingham.				
	How will it	work?	Trend analy cohort stud case studies	sis across 20 years of c ies over 7 consecutive a ; and a wider R&D port	lata; representative longitudinal academic years supported by t <mark>folio</mark> .
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COHORT STUDIES	Concep Attain Learne Classr Institu Educa Socio- Future de Early year disciplines	Infograph otual framewor ment and partic er attitudes towa toom and pedag tion factors tion system fac cultural/familial evelopment are s mathematics and in transitio	hics Statis	stical modelling Date Primary Cohort: Reception to Year 6 Secondary Cohort: Year 7 to Year 11/13 Advanced/Higher Cohorts: Year 12 to University Year 12 to University	 Ata dashboards Methodology: Pupil, teacher, leadership, parent/guardian surveys Case study class visits and interviews 400 institutions 400 institutions 4000 teachers 40000 learners

The Observatory team is keen to collaborate with others who share the same vision, making connections across the mathematical education landscape. To discuss a potential partnership with us, please get in touch at <u>MathsObservatory@nottingham.ac.uk</u>.

The Observatory at a glance

Foreword

In November 2023 we announced the Observatory for Mathematical Education. It was established with the support of a founding grant from XTX Markets and builds on a halfcentury-long tradition of internationally influential research and development in mathematical education at the University of Nottingham.

Our growing team has a simple yet ambitious goal – to undertake and disseminate an outstanding programme of large-scale longitudinal research that supports the improvement of mathematical education and delivers longterm benefits for individuals and society.



Partnering with hundreds of schools across England, UK universities and other stakeholders, the Observatory aims to become the go-to place for evidence-led systems analysis of mathematical education, from the early years to postgraduate level. We champion *mathematics-for-all* and *mathematical excellence* in an education system that equips all young people for their futures as learners, employees and citizens.

A builder once told me that he was always happier 'getting above ground' and beyond the foundation-laying stages where the unexpected often happens. During the first year of the Observatory, our growing team has encountered its fair share of surprises. However, we are now 'above ground' and making good progress on the challenging task of setting up the largest ever cohort studies of mathematical education in England.

To extend the metaphor, this report presents the *design concepts* and *preliminary plans* for the Observatory. It begins by making the case for the importance of the mathematical sciences and their applications, and for a mathematical education fit for the mid-21st century. It proceeds to explain some of the underpinning principles of the Observatory and then concludes by detailing the high-level research design, concepts and plans, focusing on our core programme of work.

In future, the Observatory for Mathematical Education will publish an annual research synthesis based on our trend analyses, cohort studies and other research and development activities. We will report on historical and emerging successes and disseminate evidence on longstanding systemic problems and the extent to which policies, interventions and practices have addressed, or are addressing, these issues. Each year we will also be producing focused reports on some of the key aspects of the mathematics education system in England.

We have been delighted with the positive response to the Observatory in this foundational stage and welcome further collaborations with practitioners, educational leaders, policymakers and stakeholders both from here in the UK and across the world.

Adrew J Noyes.

Professor Andrew Noyes December 2024

Section 1: Mathematics counts more than ever

"This is the era of mathematics and its influence will become still more intense. It is a discipline in which the UK can shine and lead. Now is the time to invest in its future in the UK." Nicholas Stern^a

Evidence Excellence Equity

Mathematics counts - more than ever

1.1 The era of the mathematical sciences

In 2018¹, Philip Bond's review of knowledge exchange in the mathematical sciences stated that we are in 'the era of mathematics'. Across the world, the mathematical sciences are increasingly used to solve problems in a range of fields that would be otherwise intractable. Without advanced mathematics, many things would simply be impossible, from mobile phones² to artificial intelligence³ to MRI scanning⁴ to genetic testing⁵. And the mathematical sciences will be key to addressing the big issues of our time, including on the climate, health, and productivity⁶.

Recent analysis highlights how the mathematical sciences are making a huge and increasing contribution to the UK economy: £495 billion in 2023, which is 20% of the UK Gross Value Added (GVA), up from 16% in 2010. A total of 4.2 million people were employed in mathematical sciences jobs, representing 13% of all employment in the UK, up from 10% in 2010^7 .

In the six years since Bond's review, much has changed including:

- A global pandemic demonstrated the importance of mathematical modelling and data literacy for the communication of important public health messages.
- The exponential growth of mis- and dis-information is highlighting the importance of understanding and critiquing numerical and data-driven arguments.
- The rapid and disruptive emergence of generative AI is raising challenging questions on the future of education, science and work.

The pace of these changes emphasises the need for improving the pipeline into the mathematical sciences, but it also highlights the pressing need for all citizens to be mathematically and data literate.

School mathematics has not, however, evolved at the same pace. Indeed, a school master or mistress of the Victorian era would recognise much of the mathematics that is currently taught in England's primary and secondary classrooms. Mathematical education today is arguably misaligned with the rapidly changing, data rich, technologically enhanced, and mathematically formatted world of the mid-21st century. We no longer need clerks who are simply efficient human calculators. Today's business 'clerks' do their arithmetic, algebra and data analysis with computers and use software to control computational machines. Our education system leaves our young people underprepared for these modern ways of working.

Furthermore, the shadow of the mathematics education of the cold war era hangs over the upper end of our school system. Following Harold Wilson's 1963 call that "to train the

⁴ https://pmc.ncbi.nlm.nih.gov/articles/PMC6526197/

¹ <u>https://www.ukri.org/wp-content/uploads/2022/07/EPSRC-050722-TheEraMathematics.pdf</u>

² https://nap.nationalacademies.org/read/25516/

³ https://www.siam.org/publications/siam-news/articles/the-mathematics-of-reliable-artificial-intelligence/

⁵ <u>https://www.livescience.com/physics-mathematics/mathematics/scientists-uncover-hidden-math-that-governs-genetic-mutations</u>

⁶ <u>https://files.eric.ed.gov/fulltext/EJ1277038.pdf</u>

⁷ For more details see <u>https://www.acadmathsci.org.uk/wp-content/uploads/2024/10/AcadMathSci-22Oct2024-Economic-Contribution-MathSci.pdf</u>

scientists we are going to need will mean revolution in our attitude to education"⁸, a core goal of upper secondary mathematics has been to produce calculus-proficient physical scientists and engineers, and A level Mathematics has not changed a great deal since then. This remains an important track for many, but for those not on that particular STEM pathway, school advanced mathematics is arguably less well suited. Meanwhile the power of computation has, in line with Moore's law, grown exponentially, and many prospective undergraduates in STEM and the social sciences are unaware of how coding, data analytics and computation will be central to their courses⁹.

Whatever changes, the need for a mathematical education system that can prepare all young people to flourish in their lifelong learning and portfolio careers, across their varied forms of societal engagement and in their everyday lives, is of paramount importance¹⁰. Such an effective system should also be producing the positive attitudes that are a powerful predictor of educational outcomes, yet which have for many years been notably poor and the subject of calls for action¹¹.

1.2 Mathematical education and society

The Observatory for Mathematical Education adopts the term *mathematical* rather than *mathematics*. For too long, mathematics education has been inward-looking and narrowly-focused. Often it appears to be hermetically sealed and distinct from the multifarious contexts and questions that would be transformed through its applications, and that are of interest to learners. Being *mathematical* encapsulates a big idea about educational purposes; it emphasises the importance of a mathematical outlook in every aspect of learning, science, work and life¹².

The Observatory's *mathematical* emphasis aligns with that of the Royal Society's Mathematical Futures Programme¹³, in which 'mathematical and data education' covers the breadth of the mathematical sciences and their applications. Importantly this is both as a distinct curriculum domain but also embedded across general, advanced, technical and vocational studies.

Our society does of course need mathematicians, but it needs many more people who approach the world mathematically, whether they be scientists, technicians, politicians, nurses, tradespeople, and so on. And every member of society, whatever their employment, needs to be able to navigate the quantitative aspects of their daily lives.

Reconfiguring mathematical education to address these issues will require an increased focus on excellence – excellence not just for the elite but for all, both in terms of experiences and outcomes. This raises important questions about equity.

⁸ Wilson, H. (1963). Labour's Plan for Science; Speech to the Labour Party Conference. Scarborough, October 1st. <u>https://blogs.bl.uk/sound-and-vision/2023/10/recording-of-the-week-harold-wilsons-1963-pledge-to-harness-the-white-heat-of-a-scientific-revolutio.html</u>

⁹ Evidence from our 2024 survey of starting mathematical sciences undergraduate students suggests that 31% are confident in coding or programming, and 41% think they will need to be good at coding or programming to do well in maths. That said even coding must navigate the AI-revolution. Either way, mathematics curriculum at every level will have to respond to technological advancement in a much more responsive way than at present.

 ¹⁰ It should be noted that although the focus herein is mathematics, or mathematical and data education, the research speaks to policy, systems and leadership across all subject areas, including those underpinned by mathematics.
 ¹¹ See, for example, the 2017 review by Professor Sir Adrian Smith on post-16 mathematics

See, for example, the 2017 review by Professor Sir Adrian Smith on post-ro mathematics
 https://assets.publishing.service.gov.uk/media/5a82d0b340f0b62305b947f0/AS_review_report.pdf
 ¹² Furthermore, it continues the tradition of the Shell Centre for Mathematical Education.

¹³ https://royalsociety.org/news-resources/projects/mathematical-futures/

Despite well-intentioned initiatives aimed at reducing inequalities, our educational system remains inequitable, and this is particularly so in mathematics. The 2023 Mathematics Pipeline report¹⁴ by members of the Observatory team showed how our current education system fails many young people experiencing socioeconomic disadvantage and how it advantages men over women¹⁵. This needs to change.

Additionally, England's educational system has favoured academic over technical and vocational pathways¹⁶ for too long. The Swiss, in contrast, have shown that it is possible to construct an educational system that achieves much greater equity and mathematical excellence across different study pathways¹⁷. Redesigning our system to achieve this is far from straightforward and raises challenging questions of curriculum purpose. For example, when should students branch into different mathematics learning pathways, and how would this impact on *excellence* (within and across pathways), *equity* (of meaningfulness) and *equality* (of access)?

These questions concerning curriculum-study-work-life choices, with their continuities and discontinuities, are complex and require scrutiny both within the education system, and across the school-to-work 'transition system'. To do so, our mathematical education system needs to develop better relationships and collaborations between policy, research and practice in order to make, and implement, better-informed policy. The Observatory for Mathematical Education will contribute to such an environment by providing a uniquely large-scale, longitudinal evidence base on the mathematical education system.

1.3 Pursuing systemic improvement

Improving mathematical education and achieving the twin aims of excellence and equity is a challenging and long-term project.

The Joint Mathematical Council of the UK was established more than 60 years ago in 1963 to improve mathematical education across the UK. In Nottingham, the Shell Centre for Mathematical Education was founded in 1967 with similar ambitions. In the six decades since, there have been many initiatives in policy, practice and research and yet many of the problems of yesteryear remain. Published timelines¹⁸ highlight the efforts that have been made, but also point to the competing and changing priorities.

The Trends in International Mathematics and Science Study (TIMSS), which last reported in 2019, presents a broadly positive picture of mathematics education in England over the past three decades¹⁹. Over this period, primary mathematics has seen considerable, and sustained, improvement, although more could be done to ameliorate attainment gaps that are established before children begin compulsory schooling and

¹⁴ Noyes, A. et al (2023). The mathematics pipeline in England: patterns, interventions and excellence. Accessed 22/10/2024 at <u>https://www.nottingham.ac.uk/research/groups/crme/documents/maths-pipeline-report.pdf</u>

¹⁵ The National Pupil Database (NPD) records sex/gender as a binary variable based on the pupil's legal sex.

¹⁶ Hodgen, J., Wake, G. & Dalby, D. (2017). *Mathematics in the successful technical education of 16-19 year olds*. The Gatsby Charitable Trust.

¹⁷ Hoffman, N. & Schwartz, R. (2015). *Gold Standard: The Swiss Vocational Education and Training System*. National Center on Education and the Economy.

¹⁸ Majewska, D., Rushton, N. & Shaw, S. (2022). <u>How did we get here?</u> Timelines showing changes to maths education in England and the United States. Cambridge University Press & Assessment; Boylan, M. Adams, G. and Birkhead, A (2023) <u>Landscaping Mathematics Education Policy</u>; Hodgen, J., Foster, C., & Brown, M. (2022). Low attainment in mathematics: An analysis of 60 years of policy discourse in England. *The Curriculum Journal*, 33(1), 5-24.

¹⁹ Richardson, M., Isaacs, T., Barnes, I., Swensson, C., Wilkinson, D. & Golding, J. (2020). Trends in International Mathematics and Science Study (TIMSS) 2019: National report for England (Research Report RR1086). Department for Education.

then are maintained or widen throughout the school system²⁰.

At secondary level, the latest Programme for International Student Assessment (PISA) outcomes in mathematics were more positive than expected and indicate a smaller pandemic-related effect than in many other educational systems. Nevertheless, they show a substantial fall compared to PISA in 2018²¹. Indeed, decades-long attempts to address the 'dip' following the transition from primary to secondary school²² have made little difference and in some mathematical topics things are even going backward²³.

England's post-16 mathematics provision continues to be an outlier²⁴ and although several researchers, including some Observatory team members, have been working on the problem of establishing better mathematical pathways since before the influential Making Maths Count report (2004²⁵), little has improved for the majority of learners. The last comparative study of adult skills (PIACC 2012²⁶) highlighted some concerning numeracy deficits in the adult population and the forthcoming publication of the latest PIACC study will show whether that situation has changed.

Our education system does, of course, do well for some of the very best mathematicians, at least those from the most advantaged backgrounds. Yet we still need more young people to have better mathematics qualifications, for these to be more aligned both to the modern world and the needs of this generation of young people, and for opportunities and outcomes to be more equitably distributed. Achieving this is contingent on the training, development, distribution and retention of sufficient numbers of high-quality teachers of mathematics, something which has been a national challenge for decades.

Stating the need for improvement is much more straightforward than identifying promising policy solutions and successfully implementing change strategies. Indeed, working within complex systems makes many of these problems 'wicked'²⁷. To make progress with such intractable problems, we need high quality evidence, better theories of change, improved implementation science, and the skilful orchestration of the best expertise the system has to offer.

1.4 On educational research

Quality research is necessary for evidence-led systemic improvement, yet the education research field often resembles a cottage industry. In the relatively short time since the field was established, many thousands of mainly small-scale studies have accrued rich evidence on the problems that concern researchers, and sometimes practitioners. While this research tapestry provides insights into mathematical education, it does not necessarily address the problems that are of greatest concern to politicians,

²⁰ Education Policy Institute (2024) EPI Annual report 2024 <u>https://epi.org.uk/annual-report-2024-disadvantage-2/</u>

²¹ Ingram, J., Stiff, J., Cadwallader, S., Lee, G. & Kayton, H. (2023). PISA 2022: National Report for England. Department for Education. Accessed 30/11/2024 at

https://assets.publishing.service.gov.uk/media/656dc3321104cf0013fa742f/PISA_2022_England_National_Report.pdf ²² Galton, M., Gray, J. & Rudduck, J. (1999). The Impact of School Transitions and Transfers on Pupil Attainment and Progress. Norwich: HMSO.

²³ Hodgen, J., Coe, R., Brown, M. & Küchemann, D. (2024). Educational Performance over Time: Changes in Mathematical Attainment between 1976 and 2009 in England. *Implementation and Replication Studies in Mathematics Education*, 4(1), 83-124.

²⁴ Hodgen, J., Pepper, D., Sturman, L. & Ruddock, G. (2010). *Is the UK an Outlier?* London: The Nuffield Foundation.

²⁵ Smith, A. (2004). Making Mathematics Count. London: QCA.

²⁶ BIS. (2013). The International Survey of Adult Skills 2012: Adult literacy, numeracy and problem-solving skills in England. London: BIS.

²⁷ Rittel, H. W., & Webber, M. M. (1973). Dilemmas in a general theory of planning. *Policy sciences, 4*(2), 155-169.

policymakers and system leaders, and the timeliness, scale and interconnectedness of research studies often doesn't reflect the real-world realities and challenges of education. Oftentimes in national discussions, questions on mathematical education arise for which there is no readily available analysis, or insufficient data.

Educational researchers typically hope that impact will build out from their own research. The Observatory, however, aims to flip this approach. Instead of 'evidence-based policy making' our focus is on 'problem-driven evidence building', where the 'problems' under consideration are systemic issues. This approach builds on the Observatory team members' longstanding experience in practice, policy work and system leadership.

To inform the Observatory's programme, understanding the system challenges from different stakeholder perspectives is vitally important. In addition, the ability to investigate different elements of the system in a joined-up way, to work across scales, draw on varied disciplinary and methodological perspectives, and to think long term are all pivotal. To achieve this the Observatory is drawing on the 'team science' thinking that is more common in STEM fields.

In recent years there has been a proliferation of education research, and a diversification of those doing it, but there remains little by way of large-scale organisation, and nothing like the investment and coordination seen in other key areas of public policy. An analysis by the Royal Society²⁸ suggests that the differential investment of public money spent on R&D from the health budget is 34 times greater than that in the education budget. This is not only a problem of scale but also of concentration and sustainability, both issues that the Observatory model seeks to address.

Given the Observatory's ambitions to support meaningful improvements in mathematical education, it is important for the programme to be informed by robust theories of change and action as well as by key ideas on the policy process. One such approach, the Multiple Streams Framework²⁹, distinguishes between the problem stream, policy stream and *political stream*, the overlapping of which creates policy windows and thereby opportunities for change. The Observatory's programme will help to identify or redefine important problems (i.e. in the problem stream) but it will also support the policy stream, distilling insights from the study of effective processes and interventions in our mathematical education system to inform the design and implementation of potential policy solutions in collaboration with other stakeholders.

²⁸ https://royalsociety.org/-/media/policy/projects/education-research/investing-in-a-21st-century-educational-research-

system.pdf ²⁹ Herweg, N., Zahariadis, N., & Zohlnhöfer, R. (2023). The multiple streams framework: Foundations, refinements, and empirical applications. In C. M Weible (Ed) Theories of the policy process (Fifth edition). New York: Routledge. pp. 29-64

Section 2: About the Observatory

"The scope of mathematical education needs to change from 'mathematics' to what we have called mathematical and data education (MDE); a combination of mathematics, statistics and data science, underpinned by computational tools." The Royal Society^b

> Evidence Excellence Equity

About the Observatory

2.1 The foundations

The system under observation

Mathematical education in England is a big project; approximately nine million students learning in around 24,500 schools³⁰ and colleges. They are taught mathematics by over a quarter of a million primary teachers and around 37,000 mathematics teachers³¹. The budget for maintained schools is around £51 billion, and mathematical education can lay claim to a good proportion of this. Moreover, there are more than a quarter of a million students studying subjects with high mathematical demands at English universities³².

Not only is the system very large, but it must also satisfy a unique set of stakeholder needs. These drivers for mathematical education, be they political, economic, scientific and societal, do not always align well. The 2023 report 'The Mathematics Pipeline in England', described these as *mathematics for citizenship*, *mathematics for employment* and *mathematics for its own sake*³³. This tri-partite view resonates with the Royal Society's triplet of General Quantitative Literacy, Domain Specific Competences and Foundational and Advanced Mathematics³⁴.

Why Nottingham?

The University of Nottingham has a long and distinguished record in advancing the learning and teaching of mathematics, in England and around the world. It began in 1967 with the foundation of the Shell Centre for Mathematical Education³⁵. The Shell Centre aimed for direct large-scale impact on classroom practice by using research results and methods in the design and iterative development of teaching and assessment materials. This resulted in a fifty-year program combining educational ambition and robust design for use by typical teachers and culminated in the 2016 award by the International Commission on Mathematical Instruction of its first Emma Castelnuovo Medal for "groundbreaking contributions that have had a remarkable influence on the practice of mathematics education"³⁶.

In recent decades it became increasingly clear to the Nottingham team that the most challenging problems in improving learning outcomes in practice lay at system level. The Shell Centre's designers knew how to enable typical teachers to teach mathematics better in well-supported environments. Yet there were not established methods of

³⁰ https://explore-education-statistics.service.gov.uk/find-statistics/school-pupils-and-their-characteristics

³¹ <u>https://explore-education-statistics.service.gov.uk/find-statistics/school-workforce-in-england#dataBlock-29fe55f4-d366-4de1-a51a-1350b94477c7-tables.</u> The estimate of 37,000 includes teachers who spend only part of their time teaching mathematics. The number of teachers spending most or all of their time teaching mathematics is arguably nearer to 30,000.

³² <u>https://www.hesa.ac.uk/data-and-analysis/students/what-study</u> In 2022/23, 253,355 students were enrolled on Mathematical Sciences, Physical Sciences, Engineering and Technology and Computing subject groups combined.

³³ Noyes A *et al*, (2023). The mathematics pipeline in England: patterns, interventions and excellence. Accessed on 22/10/2024 at <u>https://www.nottingham.ac.uk/research/groups/crme/documents/maths-pipeline-report.pdf</u> For a more detailed analysis of curriculum value positions see Ernest, P. (1991). *The Philosophy of Mathematics Education*. Basingstoke: The Falmer Press.

³⁴ Royal Society/ACME (2024) A new approach to mathematical and data education. <u>https://royalsociety.org/-/media/policy/projects/maths-futures/mathematical-and-data-education-policy-report.pdf</u>

³⁵ Many of the outstanding resources produced over decades can be found at <u>https://www.mathshell.org/</u>

³⁶ 2016 Emma Castelnuovo Medal

making this happen across an entire education system. Improvements did happen³⁷, but powerful system effects tend to undermine change. The complex dynamics of the forces involved – institutional, social and political – were not sufficiently well understood.

With the evolution of the research team at the University of Nottingham through the 'noughties', the tradition of design research intertwined with new threads of work on interventions, systemic change and on understanding the amalgam of factors that shape the mathematics education system. This included research on curriculum and qualifications reform, evaluation of national pilot projects, a clutch of European collaborative projects and a focus on mathematics pathways from 14-19. The team also continued a thread of research in further education and strengthened its work in early years, primary and higher education.

Engineering, experimenting and epidemiology

The cornerstones of the Shell Centre – Malcolm Swan and Hugh Burkhardt³⁸ – argued that one needed to change something in order to understand it. Malcolm was an outstanding designer/engineer of educational tasks and so his 'change' was at the instructional scale; Hugh pioneered a focus on strategic change³⁹. Their view is no doubt correct, but 'change' also needs to be carefully observed and understood at system level across institutions, regions, networks and the country. In so doing, one simultaneously strives to understand such educational change processes and increase the chances of future improvements.

These matters of scale and holism are at the heart of the field of implementation science and centrally important to the Observatory. With the goal of *understanding* as well as *improving* - and across multiple scales of policy and practice - the Observatory is 'for' and not just 'of' Mathematical Education. It aims for 'understanding to change' and, through its ongoing interventions and design research, 'changing to understand'.

The current trend in education research is for greater expenditure on programmes of experimental trialling. There is no doubt that this work is yielding useful insights on how effectively interventions 'treat' specific educational conditions⁴⁰. However, anyone who has been a teacher – whether in primary or secondary classrooms or university lecture theatres – knows that learning environments are always complex and dynamic with many different factors at play simultaneously. The Observatory asks, therefore, not only whether (educational) treatment A is better than B for addressing specific problem C, but also - and arguably more importantly – how might teachers and curriculum leaders combine a multitude of practices, resources, policies, interventions, enrichments activities, etc., to create good learning environments, better learner trajectories, and more positive attitudes to mathematics.

The Observatory is motivated in part by what one could call *educational epidemiology*, the Greek origins of which are *epi* (on or upon), *demos* (people), *logos* (the study of). The Observatory is undertaking the systemic *study of* (*logos*) the impact of a national system of mathematical education *on* (*epi*) young *people* (*demos*). One can draw a

³⁷ For example, in the Mathematics Assessment Project (<u>https://www.map.mathshell.org/</u>) that supported the introduction of the Common Core State Standards in the United States.

³⁸ For many years Professors Hugh Burkhardt and Malcolm Swan were the driving forces behind the Shell Centre for Mathematical Education's prolific output and international influence, with an emphasis on system and context.

³⁹ Burkhardt, H. (2009) On Strategic Design. Educational Designer, 1(3).

⁴⁰ See the 2021 <u>Review of EEF Projects</u>

parallel to the major data-driven epidemiological studies in health that have led to countless insights that have improved our lives.

Working at and across scales

From the start of Reception to the end of compulsory education and training takes fourteen years; the start of the Early Years Foundation Stage (i.e. birth) to completion of a PhD might take 24 or more years. During this time learners encounter many teachers, resources, schools, peers, changes in policy and practice, life experiences and so on. They make choices and experience successes and failures. Focusing on any one of these individuals, places or processes can generate fascinating insights but these often remain isolated, single pieces in a large multidimensional jigsaw. Through the large-scale and longitudinal analysis of learners' shifting attitudes and experiences, the Observatory will better understand the factors that shape mathematical outcomes for different learners.

A key framing concept for the Observatory is that educational systems are multiscale⁴¹ and research programmes must work across, and bridge, these scales. An educational system does not only comprise governance models, curriculum and assessment, accountability and regulation, and so on. An education system also includes the collective practices in countless classrooms, the skills and knowledge of the teacher workforce, the cognitive processes of learners, the marketplace of resources and professional learning opportunities, and so much more⁴². Scale-up projects and policy implementation programmes often fail for lack of attention to scale-bridging processes.

Inequalities and evidence of promise

The Observatory balances a concern for mathematical excellence *in educational outcomes*, and mathematical excellence *in educational experiences*. The former, *excellence in educational outcomes*, includes those with the potential to become the mathematical and data scientists of the future. The latter, *excellence in educational experiences*, is the right of all learners, future citizens and employees in every part of society for whom mathematical literacy in a data-driven world is increasingly important.

One of the key challenges that governments, schools and teachers face, is how to maximise the chances of success for learners of all backgrounds, and how to redress inequalities. Equity is another core principle for the Observatory, whose approach will identify places where inequitable outcomes are being, or have been, redressed. This will enable better understanding of the conditions of success, either in cases where attainment gaps have not grown so much, or perhaps even where they are being narrowed⁴³.

Through the analysis of trends in the administrative datasets and through interrogation of the Observatory's cohort study data, the team will identify interventions and processes showing evidence of promise that might be developed into new evaluations and trials. The team will also continue to undertake evaluations of interventions and programmes that are designed to achieve more equitable outcomes for learners of mathematics.

⁴¹ Noyes, A. (2013). Scale in education research: Towards a multi-scale methodology. International Journal of Research & Method in Education, 36(2), 101-116.

⁴² For a discussion of the primary mathematics landscape for teacher professional development, for example, see Greany, T., Noyes, A., Gripton, C., Cowhitt, T. & Hudson, G. (2023). <u>Local learning landscapes</u>: Exploring coherence, equity and quality in teacher professional development in England.

⁴³ See the <u>Opportunity Makers Report</u> from the US.

2.2 Vision, mission and aims

Vision

To become the 'go to' place for systems analysis of mathematical education with high policy-relevance, a focus on learners and teachers of mathematics, and concern for mathematics-for-all and mathematical excellence.

Mission

To generate and communicate evidence-driven, and policy-relevant research to improve mathematical education, learner outcomes and longer-term benefits for individuals and society.

Aims

To **integrate** analyses of varied aspects of the mathematical education system.

To **inform** mathematical education initiatives, interventions, and practices at multiple scales.

To **improve** engagement, progress, attainment, and participation in mathematics.

Too often education research is constrained by limitations of scale and a lack of relevance to policy and practice. From the outset, however, the Observatory team have engaged with senior sector leaders, policymakers and researchers to ensure that the design of the research programme is both high quality and sensitive to the challenges facing, and questions of interest to, different stakeholders.

The Observatory's Strategic Advisory Board⁴⁴ consists of individuals representing different types of stakeholders across government, industry, learned societies, think-tanks, education leaders and research funders. The Board will support the Observatory in maximising the research programme's influence and impact and regularly review the Observatory's progress against its goals and objectives.

Meanwhile, the quality of the Observatory's research is supported by its Research Expert Panel who offer peer-review of the research programme's design. The Panel's expertise covers quantitative and qualitative methods and spans primary to postgraduate education. The Observatory's engagement with schools is supported by teacher expert advisory panels drawn from primary and secondary schools in Nottingham and surrounding areas.

⁴⁴ Details of the membership of the Strategic Advisory Board and expert panels can be found in Appendix 5.2.

2.3 Overarching research questions

The Observatory's research programme is framed by the aforementioned big ideas of system thinking, complexity science, holism and scale, the latter both spatial and temporal. Its high-level task is to understand the system of mathematical education in order to improve it. England is the national case⁴⁵, and a very large case at that.

Written most simply, the two key research questions are as follows:

- 1. Who gets what out of mathematical education in England and why?
- 2. How might the system be adjusted to achieve better and more equitable outcomes?

Underneath that, the Observatory was established to explore a wide range of research questions and understand a variety of problems of policy and practice.

Expanding Question 1, the Observatory aims to understand:

- Patterns in learner attainment, attitudes, experiences and progression in mathematics in England and their relationship to a range of factors, including but not limited to, institution type, curriculum, teaching choices, educational interventions, home support and qualifications/study pathways.
- The extent to which these patterns vary between places (e.g., region, institution and classroom), learner background (ethnicity, gender, language, disability and socioeconomic status) and over time.
- How conceptual and theoretical insights can help to understand these patterns.

In practice these can be further broken down into a long list of general, interconnected questions. At times, specific questions surface for politicians, policymakers and other national bodies, for example on the attainment gap in the Early Years Foundation Stage (EYFS) or at Key Stage 3, the impact of a particular policy, or on the attrition of Black students through university degrees.

In taking a comprehensive approach to exploring the above questions, the Observatory team will be able to generate insights on a range of general as well as specific, time-critical questions.

With respect to Question 2 above, the Observatory team will:

- Identify, and investigate the nature of, institutions, interventions and policies through which learner attainment, attitudes and progression appears to be outlying.
- Develop possible areas for intervention that show evidence of promise for systemic improvement at different scales of implementation.
- Build understanding to support positive change in mathematical education.

The following section sets out the Observatory's programme of work that has been designed to address the above questions.

⁴⁵ The question of whether this work programme could be extended to other UK nations, or indeed elsewhere, has been raised. The team continue to consider this.

Section 3: The research programme

Evidence Excellence Equity

> "...because we can never know well enough the combination and salience of factors that are causing the school's or the system's failure, or exactly what it is that will turn things around, our best chance of success lies in addressing the problem from as many angles, levels and perspectives as possible" Mark Mason^c

The research programme

3.1 Three core strands of work

The Observatory's research programme has three core strands: 1) trend analysis, 2) longitudinal cohort studies, and 3) other research and development projects. These are supported by a wider programme of collaboration, communication, commentary and consultancy.

The sheer scale and interconnectedness of the research programme renders it unprecedented in ambition, scale and challenge. The longitudinal cohort studies in particular offer an outstanding opportunity to understand the formation and evolution of learner attitudes and identify places where practices produce improved attitudes, outcomes and participation.

England's national educational datasets are extensive and Observatory team members have undertaken analysis using these datasets over many years⁴⁶. Immediate plans are for a much more comprehensive and sustained *analysis of trends* over time, both tracking learners within the education system and in their later work and lives. This will show how progression, outcomes and post-16 participation in mathematics are patterned across schools and universities, in different places and regions, and by social backgrounds. It will show how current and historical policy interventions performed, for example:

- Has Teaching for Mastery narrowed attainment gaps in primary schools?
- Has the GCSE resit policy increased opportunities for those attaining a grade 4 by age 18?
- Have Core Maths students seen transferable benefits in their other areas of study, either in parallel or subsequently?
- How has the removal of HEI recruitment quotas impacted undergraduate recruitment to mathematical sciences?
- Are the earlier reported economic returns⁴⁷ to mathematics qualifications still present?

But national datasets are only so good. For education up to 18, they are not useful for understanding within-school versus between-school differences in progress and outcomes, and they offer little that might explain the processes and practices that contribute to better outcomes and higher post-compulsory participation. For that, data of different types generated at different scales are needed; from learners, teachers and parents, and from the policies and practices within the system.

⁴⁶ For example: Noyes, A. (2009). Exploring social patterns of participation in university-entrance level mathematics in England. *Research in Mathematics Education*, *11*(2), 167-183; Adkins, M. & Noyes, A. (2018). Do Advanced Mathematics Skills Predict Success in Biology and Chemistry Degrees? *International Journal of Science and Mathematics Education*, *16*(3), 487-502.

⁴⁷ Adkins, M. & Noyes, A, (2016). Reassessing the economic value of advanced level mathematics, *British Educational Research Journal 42*(1), 93-116. Dolton, P. & Vignoles, A. (2002). The return on post–compulsory school mathematics study. *Economica*, 69(273), 113-142.

The *longitudinal cohort studies*, together with their subset of school case studies, will generate such rich data, both quantitative and qualitative. Taken together, this interlinked, multi-level, time-series dataset will be used to investigate a wide range of questions and evidence findings at unprecedented scale. In the following subsections we explain each of the core research strands in more detail.

3.2 Trend analysis

Often analyses of trends in education focus on a particular point in the system, e.g., the number of A level entries, and observe the variation from year to year. This can highlight step-changes, such as that recently seen in A level Mathematics entries⁴⁸. However, this approach can also mask changes in the underlying pattern as the effect of one part of the education system is not viewed in the context of the preceding phases.

The Observatory team has adopted a different approach, following national cohorts of students as they move through the education system. Each cohort going through the system experiences a different combination of policies and pedagogies as strategies come and go. The effect of a change at early years level will only be felt in postgraduate education 20 years later, if at all. The team will therefore track each cohort to highlight where and when in the system things are going well and where there are challenges.

Adopting a cohort-tracking approach allows the Observatory to examine how progression and attainment is predicted by:

- Personal demographics (e.g., gender, ethnicity, disability).
- Social background (e.g., socio-economic status, region).
- Education background (e.g., prior attainment, type of institution).
- Interventions and policies (at government or institution level).

Some patterns are well established, such as lower progress by students from poorer backgrounds and higher post-16 participation by male students, but the underlying picture is much more nuanced. To address inequalities in the system, better understanding of the interaction between the different effects is needed. Consistently analysing these trends over time and finding pockets of 'over-achievement' will give clues as to which policies and interventions are making a positive difference.

To undertake this analysis, the Observatory benefits from a data lab dedicated to mathematical education research. Thanks to an Assured Organisational Connectivity agreement with the Office for National Statistics Secure Research Service, the Observatory's researchers have access to the education records of millions of individuals from 1985/86 to the present day (see Figure 1).

Combined, these form one of the richest education datasets in the world, and they give the Observatory an opportunity to analyse trends in mathematical education that does not exist in most other education systems outside England.

⁴⁸ Between 2023 and 2024, entries to A level Mathematics rose by more than 11% to over 100,000. <u>https://www.gov.uk/government/statistics/provisional-entries-for-gcse-as-and-a-level-summer-2024-exam-series/provisional-entries-for-gcse-as-and-a-level-summer-2024-exam-series</u>

Data sources

- The National Pupil Database (NPD) from the Department for Education. This contains pupil level information in the school census collected each term.
- Student data from the Higher Education Statistics Agency (HESA), supplied by each UK university each year, on courses and qualifications.
- Longitudinal Educational Outcomes (LEO) data which links NPD and HESA records to data regarding employment and earnings supplied by HMRC and the Department for Work and Pensions.

With such rich and comprehensive data, many questions can be usefully investigated with reasonably straightforward descriptive statistics. The Observatory's analysts aim to create data dashboards and powerful infographics that will communicate patterns of participation and attainment to education leaders and policymakers.



Figure 1: Data initially available for different birth cohorts from 1986/87 onwards. Data from additional cohorts and education phases will be added over the lifespan of the Observatory.

Beyond that, more advanced statistical models will be used to investigate interactions between different factors. For example, multi-level models, suitable for the nested structure of pupils within schools within regions or educational trusts, will account for the non-independent nature of the observations. Meanwhile log-linear repeated measures models can be used to investigate earnings over time, quasi-experimental designs can be adopted to evaluate the impact of particular policies and interventions with propensity scores can be used to create 'control' groups matched to 'treatment' groups.

The Observatory's systems-level approach means everything from EFYS enrolment to mid-career earnings are captured in one single dataset, enabling connections across the system to be uncovered. Furthermore, combining data from over 20 cohorts offers enormous power to detect the 'signal' amongst the very complicated 'noise' of an interconnected society where each individual's experience of school, college, university and workplace is unique.

3.3 Longitudinal cohort studies

The Observatory's primary and secondary cohort studies are similar in structure. A third cohort study bridges advanced and higher mathematical study and is, by nature, more complex with multiple choice processes, transition points and institution-types.

- The primary study follows a cohort from Reception in 2024/25 to Year 6 in 2030/31.
- The secondary study follows a cohort from Year 7 in 2024/25 to Year 11 in 2028/29 (in 11-16 schools) and to Year 13 in 2030/31 (in 11-18 schools).
- The advanced-higher study includes following a cohort of A level Mathematics students from Year 12 in 2024/25 to Year 13 in 2025/26 and then, for some, into undergraduate and postgraduate study (2026-2031).

An overview of the cohort study timeline is shown in Figure 2.

The Observatory's cohort studies will investigate key transition points: Reception to Year 1, Year 6 to Year 7, Year 11 to Year 12, and Year 13 to first year undergraduate study (or other education and employment routes). Therefore, in 2024/25, the Observatory is conducting an additional *Transition to Higher Education Study* to learn more about learner expectations and understanding of undergraduate mathematics as they transition to university study.



Figure 2: Overview of the Observatory cohort study timeline.

Conceptual framework

The Observatory's conceptual framework for the cohort studies comprises six themes:

- 1. Learner attainment and participation. This is the key area of interest in the Observatory's core programme. Administrative datasets are used to explore patterns in attainment at EYFS, Key Stage 2 and GCSE for all learners, and thereafter in advanced and higher mathematics. Patterns in post-16 participation, and in higher education choices, are also investigated.
- Learner attitudes towards mathematics. Attitude is broadly defined and comprises three key dimensions (cognitive, affective and behavioural⁴⁹); learner surveys contain self-reported items on values, confidence, enjoyment, anxiety and behaviour (intentions) about the learning of mathematics.
- 3. **Classroom and pedagogic factors**. This theme explores experiences of learning mathematics in classes/lectures including, but not limited to, classroom-level learning and teaching approaches. Learner surveys contain self-reported items on learning experiences in mathematics classrooms. Teacher surveys contain self-reported items on teaching strategies from teacher perspectives and on teacher perceptions of learning mathematics. Case studies explore the complexity of classroom practices and teacher pedagogic choices.
- 4. **Institution factors**. This theme considers variables over which institutions have some control. These include, but are not limited to, staffing (e.g., learner-to-teacher ratio; years of teaching), learning resources (e.g., textbooks, use of technologies, grouping, learner support), staff professional development, institutional priorities (e.g., funding and development foci), connections with other institutions and use of initiatives. Mathematics leadership surveys contain self-reported items on these matters. Case studies explore these issues further.
- 5. Education system factors. This theme explores factors at a policy/nationalcurricular scale, such as teacher experience and qualifications, national curriculum/assessment, the availability of general resources, professional development, networks and initiatives. These issues are explored through teacher and leader surveys and case studies.
- 6. **Socio-cultural/familial factors**. This theme considers social, cultural and familial influences on mathematics education such as home and community support for mathematics learning (e.g., private tuition, parental attitudes to mathematics, parental aspirations). Parent/guardian surveys contain self-reported items on parental perceptions of mathematics and support for the learning of mathematics.

These six themes are shown in nested format in Figure 3.

This conceptual model of the mathematics education system is nested, with classrooms and schools at its heart. There are other ways of thinking about educational systems that focus, for example, more on place-based education and/or educational networks, but these do not always take proper account of scale bridging, knowledge flows, and change logics. The Observatory longitudinal cohort studies take into consideration these different ways of mapping a national mathematics education system, acknowledging the

⁴⁹ Wen, R. & Dubé, A. K. (2022). A systematic review of secondary students' attitudes towards mathematics and its relations with mathematics achievement. *Journal of Numerical Cognition*, *8*(2), 295-325.

The research programme

complexity of the system and recognising the importance of context at all levels of a nested system.



Figure 3: Conceptual model informing the Observatory's cohort study research design.

Primary and secondary school recruitment

During 2024, the Observatory team set out to recruit⁵⁰ Research Partner Schools for the primary and secondary cohort studies and mirror the school numbers in international comparative studies⁵¹. However, in contrast to those studies⁵², the Observatory includes all learners in the cohort year group within each participating school. Another important difference is that the study groups learners into classes. The evidence is clear that teachers make a difference to learning outcomes, so understanding classroom/teacher effects, and the extent of within-school differences in attitudes, experience and outcomes, is important.

In the secondary sample, the number of selective schools and single-sex schools were fixed at 19 and 21 respectively (including 12 that are both single-sex and selective) to ensure that some were included. Despite the relatively small numbers, understanding mathematical education in these schools could provide important insights into issues of gender and socio-economic status.

	Schools	Classes	Learners	Teachers
Primary	179	>260	c. 7,200	c. 2,250
Secondary	151	>1000	c. 28,000	c. 1,500

Table 1: The sample of Research Partner Schools.

Throughout the recruitment process, the sample was monitored for representativeness using fourteen school characteristics which are the most relevant to the study. The

⁵⁰ Invited schools were provided with an information sheet and details of annual incentive payments.

⁵¹ There were 165 secondary schools in PISA 2022; 139 primary and 136 secondary schools in TIMSS 2019.

⁵² TIMSS and PISA sample within each school. The OME learner sample size is therefore considerably higher. This makes a much wider range of analysis possible.

achieved sample of Research Partner Schools can be seen in Table 1 including approximate numbers of learners and classes.

Data collection cycle

The data generation process for Research Partner Schools centres on a suite of linked surveys:

In every year:

- Pupil surveys: Paper survey of multiple-choice questions about mathematics attitudes and experiences, administered in a mathematics lesson for secondary, and either individually by an adult (Reception-Year 2) or as a whole class (Year 3-6) for primary⁵³. These are then digitised.
- **Pedagogy survey**: Online survey for the cohort's current teachers of mathematics, asking about mathematics teaching choices for the cohort classes.
- Maths Lead/Head of Mathematics survey: Online survey about the school's mathematics policies, curriculum choices and approach to professional development.

In some years:

- **Teacher survey:** Online survey for all teachers of mathematics in the school⁵⁴ which asks about qualifications, years in teaching, working pattern and professional development, undertaken in 2024/25 with new staff added in subsequent years.
- **Parent/guardian survey**: Short online survey about parent/guardian understanding of and support for their child's mathematics learning (undertaken in some years).

The annual cycle for schools is summarised in Figure 4, which includes a staff audit and class list data collection to support operational processes.

Over time, data generation 'sweeps' will be combined in a linked database⁵⁵ which can be interrogated by learner, teacher, school and school group (see Figure 5). Furthermore, this cohort data will be matched to the National Pupil Database (NPD), enabling the team to analyse data according to pupil characteristics. This will enable insights into the attainment, attitudes, experiences and progression of learners with similar characteristics, how these are affected by class-, school- and system-level factors, and where in the system changes occur. Questions might include the following:

- How are learner attitudes (e.g., enjoyment, confidence, values and anxiety) associated with their future aspirations for mathematical study?
- How do different classroom pedagogies shape learner attitudes?
- Where, and with what school policies and teacher practices, are learners from low socio-economic status areas making better than expected progress?

⁵³ In Reception, pupils will also complete a numeracy screening test to provide a mathematics attainment baseline measure.

⁵⁴ In primary schools, all class teachers will be invited to complete the survey.

⁵⁵ Whilst all Observatory work is stored in University of Nottingham secure digital environments, the bespoke database has enhanced security features and access restrictions.



Figure 4: Typical flow of activity in each of the school cohort study years.

Some of the Research Partner Schools have been invited to become case study schools. In these schools Observatory researchers will observe lessons⁵⁶ in the cohort year group, interview teachers and curriculum leaders, and meet other staff within the school. A core group of case study schools is being recruited in the first year and will be expanded in subsequent years to explore emerging issues or priority topics.

The research instruments (surveys and case studies) will inform one another iteratively, in keeping with mixed-methods research approaches. Essential to initial and ongoing instrument development are ethical principles⁵⁷ of minimising the burden on learners, teachers, parents/guardians and administrative staff in Research Partner Schools. All surveys are short and use plain language. These are framed in a way which values the work/roles of learners, parents/guardians, teachers and curriculum leaders. Participant data is pseudonymised using ten-digit Unique Participant Identifiers (UPIs) so that data is stored confidentially whilst being locatable should an individual decide to withdraw.

Advanced and higher mathematical education

The advanced-higher cohort focuses on academically demanding programmes of study⁵⁸, that is those studying A level Mathematics from age 16-18, and those choosing to start undergraduate degrees in the mathematical sciences. In the early stages of this study, the research is subdivided into three linked projects:

- 1. Students' mathematical choices (from 16-18).
- 2. Students' transitions to undergraduate mathematical sciences study.
- 3. Mapping the higher education landscape of mathematical sciences courses.

These will, in turn, form a Higher Education cohort study beginning where the 16-18 choices study concludes. There will be some following up of the future study, employment or training of the A level Mathematics students in subsequent years, but the Higher Education cohort study is intended to begin here, seeking to understand the attitudes, attainment, experiences and progression in university mathematics study.

⁵⁶ Much has been written about scientific approaches to observing and measuring instructional practice. The Observatory is drawing on the oft-used Three Basic Dimensions literature to design research instruments: Praetorius, A-K., Klieme, E., Herbert, B. & Pinger, P. (2018). Generic dimensions of teaching quality: the German framework of Three Basic Dimensions. *ZDM*, *50*(3), 407-426.

⁵⁷ The primary and secondary cohort studies received ethical approval from the University of Nottingham in March 2024 (Refs: GriptonC_268; GriptonC_269) and a programme of piloting has informed the redesign of instruments and operational processes.

⁵⁸ See <u>www.nottingham.ac.uk/observatory</u> for examples of our other research in post-16 mathematics in FE.



Figure 5: Outline of data structure generated through the Observatory cohort study methods.

Study 1: 16-18 choices study

The Observatory team is undertaking a two-year study of A level Mathematics students in Year 12 in 2024/25 and Year 13 in 2025/26. This will be completed in around 40 sixth form colleges⁵⁹ as well as the 96 Research Partner Schools with sixth form provision from the secondary cohort study. The sixth form colleges include those post-16 academies, free schools and sixth form centres with 20 or more A level Mathematics entries⁶⁰, but exclude general Further Education colleges⁶¹. This study aims to understand motivations for (de)selecting mathematical science degrees and helping to explain why undergraduate applications have remained static despite the notable rise in entries for A level Mathematics.

The data generation process for Research Partner Schools and Colleges follows the same operational protocols as the primary and secondary cohort studies and comprises of a range of surveys each year.

- **Student surveys**: Online survey of multiple-choice questions about mathematics attitudes, experiences and intentions for future study, conducted in a mathematics lesson.
- Pedagogy survey (secondary schools): Online survey for the cohort's teachers of mathematics, asking about resource and pedagogy choices (these teachers will have completed the teacher survey as part of the secondary cohort study).
- Teacher survey (colleges): Online survey for teachers of advanced mathematics combining questions on their resource and pedagogy choices, as well as on their qualifications, years in teaching, working pattern and professional development.
- Head of Mathematics survey: Online survey about the school's/college's mathematics policies, curriculum choices and approach to professional development.
- **Parent/guardian survey**: Short online survey about parent/guardian understanding of and support for their child's mathematics learning (undertaken in Year 12 only).

In Year 13, we will invite the students to continue participating in the cohort study and complete follow up surveys in subsequent years about what they do next including any who continue to undergraduate study in mathematical sciences.

Study 2: Transition to higher education study

In September 2024, the full cohort of 6857 individuals about to embark on undergraduate mathematical science degrees in universities across the UK were invited to participate in an online survey. This explored their qualifications and attitudes towards mathematics, their understanding of the nature of mathematics and their perspective on how they learn mathematics. 1003 students completed this initial survey and will be invited to complete a second survey in February 2025 about their initial experiences of their courses, repeating questions about attitudes, values and learning. Data will be analysed by university type, entry qualifications, individual characteristics and home/international status. It will also be used to understand retention patterns.

⁵⁹ Recruitment of Research Partner Colleges has recently opened with all 122 eligible institutions invited to participate.

⁶⁰ Based on 2022/23 data from the Department for Education in England.

⁶¹ Observatory staff recently conducted a Nuffield funded project on <u>Mathematics in Further Education Colleges.</u>

Study 3: Higher education landscape study

The Higher Education landscape is rich in the diversity of provision, with each university free to set its own curriculum, teaching philosophy, assessment regime and degree standards. This gives students huge levels of choice but makes it much harder to fully understand the different forces behind patterns of participation and attainment. In addition to education factors, many students also face challenges around living independently, managing student loans and part-time work, and changes to their social support structures. These challenges are not unique to mathematical sciences students but may confound our analyses if not accounted for. An added complexity is the different routes students can take through the system, with many courses making provision for additional foundation years, years abroad or in industry, or an undergraduate Masters year.

Before we commence the Higher Education cohort study (see next subsection) we first need to understand this complex landscape of mathematical provision. Our initial work will develop a typology of mathematical sciences courses looking at different dimensions such as:

- Mathematical theory (e.g., lemmas and proofs) vs applications of mathematics.
- Didactic pedagogical techniques (e.g., lectures) vs active learning pedagogies.
- Traditional closed-book exams vs extended project-based assessments.
- Curriculum provision in areas such as calculus, algebra, probability, etc.

We will investigate whether the above dimensions are correlated with each other or whether they relate to other factors such as entry tariff. Importantly, we will develop the typology using the information supplied to prospective students, i.e. the language mathematics departments use to describe their courses, to examine any relationship with the types of students who later enrol on each course. The subsequent cohort study will detect any notable inconsistencies between marketing materials and reality. There are over 850 undergraduate courses in the UK where mathematical sciences comprise at least half the curriculum, so the typology will focus on BSc Mathematics courses, with the option to add in specialised or joint honours courses later.

Higher education cohort study

This aspect of the work programme is under development and will be contingent upon the three initial projects outlined above. The initial plan is to invite all first-year mathematics undergraduates in participating universities in 2026/27 to complete an online survey about their attitudes to mathematics and experiences of mathematics learning. These students will then be followed up in each subsequent year of their course. In a small number of case study universities, observations and interviews with undergraduate and postgraduate students will be conducted, as well as with course leaders and lecturers. Selection of participant and case study universities and research instrument design will be informed by the findings of the Observatories previous trend analysis and projects in this area.

3.4 The wider research and development portfolio

Since the Shell Centre for Mathematical Education was founded in 1967, at least £40 million (at today's prices) has been invested in research and development projects in mathematics education at the University of Nottingham. This includes dozens of

projects, funded by many different organisations, undertaken both nationally and internationally.

This portfolio of projects includes many that have taken an engineering approach⁶² to the design of education materials, together with the more recent growth of evaluation, experimental and observational research studies.

In time, the Observatory team will publish a comprehensive account of the history of the research in mathematical education at the University of Nottingham⁶³. Details of two current EEF-funded research trials are below.

Mastering Maths

Mastering Maths is a large-scale randomised controlled effectiveness trial, funded by the Department for Education's Accelerator Fund and commissioned by the Education Endowment Foundation. It is investigating the extent to which the Mastering Maths professional development programme enables teachers to apply new teaching approaches and the subsequent impact on attainment outcomes for GCSE Mathematics resit students⁶⁴. The professional development programme is the outcome of intensive and substantial research carried out as part of the Observatory team's earlier work on the Centres for Excellence in Maths programme led by the Education and Training Foundation over the period 2018-2023.

Central to the carefully designed intervention programme are five key principles in Teaching for Mastery in Further Education which are exemplified in a handbook⁶⁵ that was produced collaboratively by stakeholders including teachers in FE. The professional development programme involves teaching lessons based on these mastery principles and engaging in a lesson study process in geographical clusters.

The programme builds on an efficacy trial conducted in 2021/22 which found that the intervention had a positive impact on students equivalent to one month's extra learning. Even more encouraging was that for students eligible for free school meals this learning gain was equivalent to two months. This is even more remarkable considering the GCSE resit course lasts only around eight months.

The intervention will take place throughout the academic year 2024/25 and the evaluation will be undertaken by independent evaluators NatCen (National Centre for Social Research).

Counting Collections

Counting Collections is a hands-on early mathematics approach to develop children's number sense (understanding of number and quantity), developed by Observatory staff for use with children in reception classes (four- to five-year-olds). The Education Endowment Foundation commissioned a randomised controlled efficacy trial of the approach between January 2023 and April 2025 with 90 teachers randomised to the intervention group and 90 to the control group.

⁶² Burkhardt, H., & Schoenfeld, A. H. (2003). Improving Educational Research: Toward a More Useful, More Influential, and Better-Funded Enterprise. *Educational Researcher*, 32(9), 3-14.

⁶³ Details of some of the more recent, relevant and influential projects can be found at

https://www.nottingham.ac.uk/observatory/projects/legacy-research-projects.aspx 64 The project builds on Malcolm Swan's Standards Unit work: Swan, M. (2005). *Improving learning in mathematics*.

⁵⁴ The project builds on Malcolm Swan's Standards Unit work: Swan, M. (2005). Improving learning in mathematics. Department for Education and Skills.

⁶⁵ https://www.et-foundation.co.uk/wp-content/uploads/2020/03/CfEM_Mastery_Handbook.pdf

The research programme

The Counting Collections approach supports counting, subitising, comparing numbers and composition of numbers. It involves children using containers of objects (manipulatives) to find how many are in the collection. Children work in pairs to choose, strategize, count and record how many items there are. They do this in weekly sessions of about 30 minutes.

The intervention also involves adding a Counting Collections area to the classroom's continuous provision. This is a counting library of boxes of different sized collections of inviting objects to count, as well as tools to aid counting such as ten frames, mats, pots, numerals and number tracks. Children can access this provision during play, providing additional opportunities for number assessment and interaction.

Schools are supported to implement the programme through teacher professional development including in-person and online sessions, and through an online learning platform. The trial is being evaluated by Sheffield Hallam University who will compare attainment data for the number learning of the children in the intervention and control group classes. They will also conduct an implementation and process evaluation of the intervention.

In September 2023 to March 2024, the Education Endowment Foundation provided additional funding to support the development of Counting Collections (Preschool) for practitioners working with two- to four-year-old children in early years settings. This research was conducted in partnership with Heart Early Years Stronger Practice Hub in the West Midlands.

3.5 Future developments

Over the last year the Observatory's plans have been presented to many people, including civil servants, policy advisors, senior stakeholders, researchers and teachers. These discussions considered the alignment between planned work and the needs for evidence on a range of issues of importance to these and other stakeholders. These meetings have generated four key areas for future expansion of the Observatory's portfolio. In most cases these are areas in which the team already has experience and expertise.

Early years maths

This is a priority area for the new government and for tackling the education inequalities that blight our system. The Observatory's primary cohort study and Counting Collections are already working in this space, and funding for a larger scale trial of the latter intervention with three- to four-year-olds is being sought.

With close connections to the Centre for Early Mathematics Learning at the University of Loughborough, and through advising government on their flagship mathematics programmes for birth to five years through the Early Childhood Mathematics Group, the Observatory is keen to apply its systematic, multi-scale approach to improving early learning of mathematics.

Initial teacher education

Attracting and supporting new mathematics teachers through their initial training and early career phase is a critically important part of national agendas for mathematical excellence, as too is the development of generalist primary teachers' mathematics subject knowledge and pedagogical content knowledge. Furthermore, the support for out-of-field teachers, whether in secondary or further education is a national concern and longstanding political priority.

The Observatory's cohort studies will give a partial view on some of these issues and build on previous work⁶⁶. However, the relationships between teacher supply, the effectiveness of training, and teacher distribution and matters of mathematical excellence and equity are closely entangled and the Observatory team intend to expand the work programme to include a project on teacher education.

Mathematics in FE, across the disciplines, and in transitions

Several members of the Observatory team were central to the development of the General Mathematical Competences framework⁶⁷ that informed the T level initiative. This built on many years of involvement in designing, advising on, and evaluating Free Standing Mathematics Qualifications, Use of Maths and Core Maths. This and other research on mathematics in the FE sector⁶⁸ is an area of strength for the team, as is the role of mathematics and data across the disciplines and at the transition to university.

The Observatory hopes to extend the secondary cohort study into all of the various study pathways when students get to 16. In the meantime, we will continue to develop interventions, research and evaluate, and advise on policy decisions in this space. We are keen to establish a far more comprehensive programme of research on mathematical and data learning across disciplines and programmes post-16 and into university.

Quantitative, statistical and data literacies

The Royal Society recognise there is a "need for all students to confidently apply their mathematical and data skills to common, real-world, quantitative problems in a range of educational, employment and everyday contexts"⁶⁹. Equipping all individuals to use their mathematical understanding to tackle realistic, open-ended problems and to communicate using numbers and statistics is vitally important in a data-driven society.

Observatory team members have been involved in writing curriculum statements for mathematical literacy courses and have considered assessment design in this context⁷⁰. The Observatory's cohort studies will give some insight into how mathematical literacy is being supported in primary and secondary education but the team hopes to expand this to consider the development of quantitative and data literacy across subjects and into adult life.

⁶⁶ Noyes, A., Dalby, D. & Lavis, Y. (2022). Mathematics in England's further education colleges: who is teaching what, and why it matters. *Journal of Further and Higher Education*, 46(10), 1347-1361.

⁶⁷ https://royalsociety.org/-/media/policy/topics/education-skills/maths/mathematics-tlevels-gmc.pdf

⁶⁸ For example the <u>Mathematics in Further Education Colleges</u> project and Centres for Excellence in Mathematics programme (<u>CfEM</u>).

⁶⁹ Royal Society/ACME (2024) A new approach to mathematical and data education. <u>https://royalsociety.org/-/media/policy/projects/maths-futures/mathematical-and-data-education-policy-report.pdf</u>

⁷⁰ For example, in the development of Free-Standing Mathematics Qualifications, AS Use of Maths and Core Maths.

Section 4: Everyone counts

"There's barely any aspect of our modern lives that hasn't had a mathematical contribution at some point and yet, if you asked the average person, they might think that maths is just difficult, irrelevant and uninteresting" Hannah Fry^d

Evidence Excellence Equity

Everyone counts

4.1 A complex educational landscape

Alongside tens of thousands of nurseries, schools, colleges and universities, there is a complex patchwork of over 140 organisations that are researching, enabling and shaping mathematics education in England.

This complexity is highlighted in Figure 6 in which the varied parts of the landscape are organised into themes, most of which are listed below. Greater detail is shown in Figures 7, 8 and 9 in Appendix 5.1^{71} .

A remote observer of the system will see:

- Learned societies promoting mathematics and the profession.
- Educator networks sharing experiences and ideas amongst practitioners.
- Research associations advancing and disseminating research.
- Research groups and institutes studying different aspects of the system.
- Government departments and regulators meeting political objectives.
- Funders supporting research and initiatives in line with their criteria.
- Awarding bodies upholding qualification standards.
- Resource providers developing teacher aids and learner experiences.
- Intervention providers supplementing mainstream education.
- National bodies coordinating different aspects of the system.

Given the varied demands on mathematical education, the motivations and agendas of these different types of stakeholders are many, and there is further variation in outlook between the organisations within each of these categories. Sometimes these organisations, like individuals in any education setting or workplace, work harmoniously to achieve shared or complementary aims, while at other times they pull the system in different directions. This tension, which can be constructive, needs to be navigated by anyone seeking to change the system.

The Observatory aims to evaluate the effectiveness of how well the system, with its many players and drivers, is currently functioning. Developing a holistic view of the positions, relationships, influences and activities of different stakeholders is not straightforward, but it is necessary in order to improve change planning.

⁷¹ It should be noted that this analysis is not exhaustive; any omissions are unintentional. Furthermore, the uniformity of sizes does not reflect the varied scale and reach of these organisations. Thirdly, this representation makes no attempt to map the relationships between the organisations and programmes. Comments, additions and corrections are welcome to <u>MathsObservatory@nottingham.ac.uk</u>.

Everyone counts



Figure 6: Mathematical education landscape in England – key components^{72 73}.

4.2 Collaboration

The mathematical education system is shaped by everyone, from the Secretary of State to the newly qualified teacher, from the learned professor to the primary school pupil. In this system, everyone counts. No single organisation can fully comprehend this complexity, but the Observatory aims to collaborate with others who share its vision for evidence-informed excellence and equity, thereby strengthening and connecting different pieces of the whole.

The Observatory will draw attention to both areas of strength and improvement within the system and is keen to partner with other organisations to effect change. The team already has valuable connections with some of the organisations shown in the mathematical education landscape in England (Figure 6), some of which are

⁷² See the Observatory website for a fully labelled version of the diagram –

https://www.nottingham.ac.uk/observatory/documents/reports/maths-ed-organisations-uk-dec-24.pdf

⁷³ Components of the diagram are included in the Appendix 5.1 of this report.

represented on our advisory groups (see Appendix 5.2), but it is also actively seeking opportunities to collaborate with others.

The Observatory invites all who want to work in partnership to improve mathematical education to get in touch. This includes, but is not limited to:

- Educators with experience of practices that do or do not work.
- Educational leaders with questions or suggestions about best practice.
- Policy makers wanting trustworthy answers to pressing questions.
- Educational charities seeking to increase or evaluate impact.
- Researchers undertaking complementary research.
- Funders interested in knowing more about the Observatory's capabilities.

4.3 Conclusion

The Observatory is embarking on an exciting new chapter of mathematical education research in England. Building on the strong foundations of their predecessors, the Observatory team have a tremendous opportunity to leverage new insights into who thrives in mathematics and why. Using 20 years' worth of government data, the research will reveal the details of patterns in student attainment and participation. Furthermore, case study visits and the responses from students, teachers and parents across primary, secondary and tertiary education will shed new light on the reasons behind those patterns. These longitudinal cohort studies are unprecedented in their scale and ambition. However, the programme outlined in this report is not one of research for research's sake.

The Observatory's stated mission is to generate and communicate state-of-the-art, evidence-driven, and policy-relevant research to improve mathematics education, learner outcomes, and longer-term benefits for individuals and society. Whilst the team that has been assembled and the programme of research that has been started will almost certainly be able to generate and communicate relevant research, making any significant improvement to mathematics education is a far greater challenge. Working in partnership with individuals and organisations across the mathematical education landscape, the Observatory seeks to fulfil its vision of mathematical excellence in educational outcomes for all those who have the capacity to achieve this, and mathematical excellence in educational experiences for all children and young people.

Section 5: Appendices

"We will always have STEM with us. Some things will drop out of the public eye and will go away, but there will always be science, engineering, and technology. And there will always, always be mathematics" Katherine Johnson^e

> Evidence Excellence Equity

Appendices

5.1 Mathematical education landscape schemas





Appendices



Figure 8: Mathematical education landscape schema – examples of research associations, research institutes, research groups & centres, and funders.



Figure 9: Mathematical education landscape schema – examples of awarding bodies, government & policy organisations, education interventions, and educational resource providers.

5.2 Advisory groups

Strategic Advisory Board

Alison Etheridge, OBE	Professor of Probability, University of Oxford; President, Academy for the Mathematical Sciences, FRS.
Charlie Stripp, MBE	CEO Mathematics in Education & Industry (MEI), Director of the National Centre for Excellence in the Teaching of Mathematics (NCETM).
Darra Singh, OBE	Strategy Director Newton, Board member at Bradford Children and Families Trust.
Hardip Begol, CBE	Board Member at Ofqual, the Oak National Academy and the Education Policy Institute.
Jeremy Benson	Deputy Director, STEM Education and Curriculum Strategy, Department for Education, FRSPH.
Josh Hillman	Director of Education, Nuffield Foundation.
Lucy Marie Hagues, MBE	CEO Capital One UK (Chair of the SAB).
Peter Finegold	Head of Policy Education and Skills, The Royal Society, FRSA.
Rebecca Boomer-Clark	CEO Lift Schools (previously Academies Enterprise Trust).
Simon Coyle	Head of Philanthropy, XTX Markets.

Research Expert Panel

Alf Coles	Professor of Mathematics Education, University of Bristol.
Ann Dowker	Senior College Lecturer in Experimental Psychology, University of Oxford.
Camilla Gilmore	Professor of Mathematical Cognition, Loughborough University.
Cathy Smith	Professor of Mathematics Education, Open University.
Christian Bokhove	Professor of Mathematics Education, University of Southampton.
Jake Anders	Professor of Quantitative Social Science, UCL.
Jenni Ingram	Professor of Mathematics Education, University of Oxford.
Matthew Inglis	Professor of Mathematical Cognition, Loughborough University.
Peter Rowlett	Reader in Higher Education Mathematics Teaching and Learning, Sheffield Hallam University.
Rachel Marks	Associate Professor of Mathematics Education, UCL.
William Browne	Professor of Statistics, University of Bristol.

Primary Expert Advisory Panel

Cat Stone	Primary Lead Practitioner for Mathematics at L.E.A.D Academy Trust.
Jane Gill	Primary Advisor and Mathematics Consultant at Nottingham City Council.
Jenny Murphy	Head Teacher at Hunloke Park Primary School; Primary Advocate Headteacher at East Midlands West Maths Hub.
Lorna Dermody	Head of School at Mellers Primary School.
Matthew McArthur	Mathematics Lead at Southwold Primary School.
Phil Herd	Associate Headteacher and Digital Lead at Transform Schools Trust.
Tom Isherwood	Lead Practitioner for Mathematics at Flying High Trust; Primary Assistant Maths Hub Lead at East Midlands West Maths Hub.

Secondary Expert Advisory Panel

Dominic Hudson	Director of Education at QEGSMAT.
Gemma Heald	Senior Assistant Headteacher at Two Counties Trust.
James Thomas	Rushcliffe Spencer Academy; Secondary Assistant Maths Hub Lead at East Midlands West.
Joel Haigh	Mathematics Curriculum Lead at Alderman White School.
Kate Auld	Secondary Assistant Maths Hub Lead at East Midlands East Maths Hub.
Lucy Lycett	Maths Hub Lead at East Midlands West Maths Hub.
Matilde Warden	Head Teacher at Belper School; Secondary Advocate Headteacher, East Midlands West Maths Hub.
Nicholas Wilson	Director of Mathematics at Archway Learning Trust.
Rachel West	Maths Hub Lead at East Midlands East Maths Hub.
Louise Elder	Senior Director of Mathematics at Archway Learning Trust; Secondary Assistant Maths Hub Lead at East Midlands East Maths Hub.

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- Marc North
- Michael Adkins
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