

**MODELLING BIOLOGICAL MACROMOLECULES IN SOLUTION:
THE GENERAL TRI-AXIAL ELLIPSOID**

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Appendix II Illustration of use of the Λ function (equation 50) by application to data available for the tryptic subfragments of fibrinogen

It is apparent from Figure 17 that, until the harmonic mean relaxation time τ_h can be measured to a precision greater than that currently available ($\sim \pm 3\%$ at best, assuming no significant internal rotations of the chromophore or segmental rotations of parts of a macromolecule relative to other parts), use of Λ will generally be restricted to prolate ellipsoidal particles above an axial ratio of about three.

Unfortunately, there is at present a lack of reliable steady state fluorescence depolarization data for macromolecules in this axial range. Use of the function may however be illustrated by application to data available for the tryptic fragment of bovine fibrinogen. By using a steady-state fluorescence - depolarization technique, Johnson & Mihalyi (1965) reported a harmonic mean relaxation time for fibrinogen of 195 ± 5 ns, a value lower than the corresponding value for a sphere of the same volume (299 ns); the value for τ_h of the tryptic subfragment was 178 ns, strongly suggesting that the tryptic subfragments had rotational freedom within the fibrinogen molecule. Assuming there is still no further internal rotation within the subfragment itself, one can combine this result with viscosity and molecular-weight data obtained previously by Mihalyi & Godfrey (1963).

Taking M_r as $95,000 \pm 2,000$, $[\eta]$ as (7.18 ± 0.07) ml.g⁻¹ and assuming a ± 5 ns standard error in τ_h , Λ is calculated to be 4.74 ± 0.17 where the method for calculating the standard error in Λ is given by Paradine & Rivett (1960). This corresponds from Figure 17 to a prolate ellipsoid of axial ratio 6.8 ± 0.3 consistent with the estimates of the axial ratio

derived from four other hydrodynamic parameters, three of which assume no particle swelling due to solvent association (Table 23). The results from electron microscopy studies suggest however that the subfragments are nearly spherical (Hall & Slayter, 1959); as Mihalyi & Godfrey (1963) have previously stated, this difference is probably too large to be explained by drying effects alone. At least part of this difference can, however, be possibly ascribed to an apparent discrepancy between the viscosity data of their Figure 4 with the sedimentation data of their equation 2; the latter suggests a sedimentation regression coefficient, k_s , of ~ 3.6 (after correction to solution density; Rowe, 1977), whereas the viscosity regression coefficient, k_η , is only ~ 2.5 . Rowe (1977) has shown that the ratio k_η/k_s is equal to the swelling ratio $\bar{v}_s/\bar{v}$, where $\bar{v}_s$ is the swollen specific volume in solution. Mihalyi & Godfrey's (1963) data apparently gives a value for the swelling of less than 1, indicating the particle to contract in solution, an unlikely event. Unfortunately, although the pH values of the solutions used for the sedimentation and harmonic mean relaxation time measurements are given and are near (6.5 and 7.1 respectively), that for the viscosity is not given, so this is a possible source of error.

It is hoped that the availability of the new Λ function will encourage the production of more reliable data in order to resolve these difficulties, and also accelerate improvement in the methodology so that τ_h/τ_o can be measured with much greater precision, enabling application of the Λ function to prolate ellipsoids of axial ratio less than three and also to oblate ellipsoids.

Table 23. Hydrodynamic parameters and axial ratios for the tryptic subfragments of fibrinogen

Hydrodynamic Parameter	Derived Axial Ratio	Reference
v^*	7.8	Mihalyi & Godfrey (1963)
f / f_0^*	7.1	"
β	9.3	"
τ_h / τ_0^*	5.0	Johnson & Mihalyi (1965)
Λ	6.8	This study

* Assuming no particle swelling due to solvent association

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